1961

# Jevonian value theory: A preface to marginal utility analysis

Donald Preston Cole
*The University of Montana*

# JEVONIAN VALUE THEORY:
# A PREFACE TO MARGINAL UTILITY ANALYSIS

by

DONALD PRESTON COLE

A.B. Drew University, 1959

Presented in partial fulfillment of the requirements

for the degree of

Master of Arts

MONTANA STATE UNIVERSITY

1961

Approved by:

Richard E. Sherman
Chairman, Board of Examiners

Ellis Waldron
Dean, Graduate School

MAY 19 19[illegible]
Date





ProQuest LLC.
789 East Eisenhower Parkway
P.O. Box 1346
Ann Arbor, MI 48106 - 1346

6-7-61

To D. F.

Well, why do we study the history of any science? Current work, so one would think, will preserve whatever is still useful of the work of preceding generations....It is certainly better to scrap outworn modes of thought than to stick to them indefinitely. Nevertheless, we stand to profit from visits to the lumber room provided we do not stay there too long.

Joseph A. Schumpeter

TABLE OF CONTENTS

# CHAPTER I

## Jevons in Perspective

It is the curious habit of many students of economic theory to seek out those past economists in whose writings might be found the first embryos of a given theory and to impute to such men the credit for having founded an entire school of thought. Thus we are wrongly informed that Adam Smith "discovered" economic science, that the supply-cost and marginal utility approaches to value theory were originated by David Ricardo and William Stanley Jevons, respectively, and, finally, that Alfred Marshall, using his famous scissors, was the first to cut the value problem into two parts. Those who offer such unscholarly generalities appear to ignore the fact that economic ideas, like bedrock in a vast river, have come into being only through endless evolution, each generation adding its own new layer to the hardpan. But economic science seems particularly amenable to the vague syllogizing of students largely because of its close alliance with common-sense reasoning.[1] It is this alliance which poses a barrier to

[1]See J. A. Schumpeter, History of Economic Analysis, New York Oxford University Press, 1954, p. 9.

our determining who the real originators are. We know, for example, that for many centuries laymen have intuitively understood the laws of supply and demand, and this makes it even more difficult to trace the fountainhead of supply-demand analysis. Who would venture to say that the writings of a 14th century millwright or wool merchant constitute the discovery of the theory of supply and demand? It is the old case of "cogito, ergo I am an economic theoretician."

Such is the problem which confronts us in evaluating the contributions of William Stanley Jevons (1835-82) to the theory of value: how to adjudge the originality of these contributions in light of the achievements of his precursors and the criticisms of his successors.

Our purpose is not to discount the importance of the original ideas which Jevons conceived, but rather to ferret out those for which he deserves logical priority and to set them apart from other concepts which are defective in terms of modern theory. This task is made no less difficult by the fact that many of Jevons' more original concepts are cloaked in loose, quasi-mathematical garb which does much to obscure their intended meanings. Then too, Jevons lacks the literary facility of authors such as Edgeworth and Marshall;[2] this also serves to discount his importance in

---

[2]Jevons never held any delusions about his inadequate writing ability, though even this surpassed his competence

the history of economic doctrine, and has even led contemporary writers to misjudge his achievements.

Quite naturally, Jevons' writings reflect the influence of certain intellectual ancestors,[3] the most significant of which are Bentham, Cournot and various early pioneers of marginal utility analysis. Though not properly classified as an economist, Jeremy Bentham (1748-1832), one of the first English utilitarians, had a profound effect upon the writings of many nineteenth century economists. The leader of a group known as the "Philosophical Radicals," Bentham sought to develop a science of human behavior applicable to politics, jurisprudence, economics, religion and other related disciplines. The major tenet of his philosophy--"the greatest happiness of the greatest number, that is the measure of right and wrong"[4]--is known to all students of economic history and is eloquently phrased in the first paragraph of his Introduction to the Principles of Morals and Legislation:

---

as a teacher. In his later years he wrote: "Sometimes I have enjoyed lecturing, especially in logic, but for years past I have never entered the lecture room without a feeling probably like that of going to the pillory" (Quoted in J. M. Keynes, Essays and Sketches in Biography, New York, Meridian Books, 1956, p. 159).

[3]See T. W. Hutchison, A Review of Economic Doctrines, 1870-1929, Oxford, Clarendon Press, 1953, p. 14.

[4]Jeremy Bentham, A Fragment on Government and An Introduction to the Principles of Morals and Legislation, ed. Wilfrid Harrison, Oxford, Basil Blackwell, 1948, p. 125.

> ...Nature has placed mankind under the governance of two sovereign masters, pain and pleasure. It is for them alone to point out what we ought to do, as well as determine what we shall do...Every effort that we can make to throw off our subjection will serve but to demonstrate and confirm it...The principle of utility recognizes this subjection and assumes it for the foundation of that system, the object of which is to rear the fabric of felicity by the hands of reason and law. Systems which attempt to question it, deal in sounds instead of sense, in caprice instead of reason, in darkness instead of light.[5]

This principle of utility, or, as it is popularly termed, "hedonism," presupposes that all feelings are capable of scientific measurement, so that Bentham's utility emerges as some sort of "psychic reality" which can be known by simple introspection.

Jevons is disturbed by Bentham's cardinality assumption, and tries to redefine utility in non-cardinal terms. In lieu of Bentham's proposition that utility is directly measurable, he offers the argument that utility can be measured <u>indirectly</u> by the observable effects of a feeling, as consumers react to price changes. But what he fails to realize is that this also presupposes cardinal measurement.[7] (In 1890 Marshall made a similar mistake by defining utility in terms of consumer income.) The cardinality question was a nut

---

[5] <u>Ibid</u>.

[6] See W. S. Jevons' <u>Theory of Political Economy</u>, New York, Kelley & Millman, 1957, p. 12.

[7] For an excellent statement on cardinal vs. ordinal measurement see Schumpeter, <u>History</u>, pp. 1060-1066.

which Jevons was never quite able to crack, though it must be admitted that he made an attempt in the first edition of his Theory.[8]

Modern theory has proven that the alliance between Benthamite hedonism and economics was, at best, an unholy one. Yet we must not overvalue the debt which Jevons owed to Bentham. Contemporary writers, in an apparent attempt at skimming the cream from the milk, suggest that Jevons' theories can be divorced

> from their...hedonistic basis without substantial alteration of their essential features, and with a distinct gain in the favor of actuality.[9]

Jevons himself denies that the subjective valuations of one person can be compared with those of someone else:

> ...I see no means by which such comparison can be accomplished....But even if we could compare the feelings of different minds, we should not need to do so; for one mind only affects another indirectly. Every event in the outward world is represented in the mind by a corresponding motive, and it is by the balance of these that the will is swayed....Each person is to other persons a portion of the outward world....Thus motives in the mind of A may give rise to phenomena which may be represented by motives in the mind of B; but between A and B there is a gulf. Hence the weighing of motives must always be confined to the bosom of the individual.[10]

Jevons' allusion to the problems of interpersonal utility

---

[8]See p. 18.

[9]A. A. Young, "Jevons' Theory of Political Economy," American Economic Review, II, 1912, p. 578.

[10]Jevons, Theory, p. 14.

comparison provides a key to his analysis. Whereas Bentham had inferred that such comparison was possible and, indeed, necessary for the purposes of analysis, Jevons makes an attempt to avoid this. He also tries to free himself from the ethical questions which were posed by Bentham and other early hedonists. To the extent that he was successful in overcoming these two factors, we are able to omit the utilitarianism from his writings without appreciably injuring their scientific content.

Of no lesser importance was the debt which Jevons owed to Augustin Cournot (1801-77). A French writer who pioneered econometric method, Cournot was not widely read by English economists until the latter part of the nineteenth century. One of the first English acknowledgements of Cournot's major work, Recherches sur les Principles Mathématiques de la Théories des Richesses, appears in the second edition of Jevons' Theory (1879):

> ...This work must occupy a remarkable position in the history of (mathematical economics). It is strange that it should have remained for me among Englishmen to discover its value....I procured a copy of the work as far back as 1872, but have only recently studied it with sufficient care to form any definite opinion upon its value.[11]

---

[11] Ibid., pp. xxix-xxx. After the writings of Cournot, Dupuit, and others were unearthed, Jevons felt that all questions of priority had finally been solved: "...these questions are really of little or no importance to us now ....We are all shelved on the matter of priority, except, of course, as regards details and general method of exposition, etc." (Letters and Journal of W. Stanley Jevons, ed. H. A. Jevons, London, MacMillan & Co., 1886, p. 409)

Many of the changes and corrections which Jevons undertook in his second edition, especially as regards the notation of marginal utility, might be attributed to the influence of this earlier writer. While admitting that Cournot's Recherches was so significant that it should be known to all, Jevons confessed that his own knowledge of mathematics precluded a complete understanding of its contents. It cannot be sufficiently stressed that Jevons was never the mathematician he hoped to be, as can be seen in his treatment of simultaneous equations[12] in Chapter IV of the Theory.[13] It is only unfortunate that he was so poorly equipped to handle the differential calculus which he considered so essential. As Young points out, Jevons' "use of the mathematical method is more apparent than real."[14] Aside from his poor understanding of mathematics per se, Jevons also misunderstood the real nexus between mathematics and economics. According to Eckard, Jevons apparently believed that

> ...economic relationships follow mathematical laws through an intricate succession of relationships. I grant that mathematical symbols are a very convenient aid in expressing the results of reasoning. But I refuse to concede that a mathematical operation can be validly

---

[12]For a discussion of the meaning of simultaneous equations and their importance to economics see Schumpeter, History, p. 970.

[13]See Jevons, Theory, pp. 114-118.

[14]Young, "Jevons' Theory," p. 588.

> employed to prove an economic proposition, even though it may make an excellent illustration.[15]

The general result is that Jevons, the mathematician, frequently outdistances Jevons, the economist, and sound analysis is sometimes left to fend for itself.

Jevons was not the first major writer to treat marginal utility. Others before him, notably Senior, Whately, Lloyd,[16] and Gossen, had broken ground for this conceptual device; what is remarkable is that only a handful of these men--Gossen and Senior were among them[17]--were known to Jevons. In the preface to the second edition of the Theory he acknowledges several unnoticed writers for having influenced his thought: Dionysius Lardner, whose treatment of demand in the Railway Economy served as a model for Jevons' "laws of utility"; Richard Jennings, whose Natural Elements of Political Economy provided Jevons with insights into utilitarian measurement; and, of lesser influence

---

[15] E. W. Eckard, Economics of W. S. Jevons, Washington, D. C., American Council on Public Affairs, 1940, p. 94.

[16] These first three writers represent the so-called "Oxford Utility School," which flourished during the 1830's. Prior to the appearance of Jevons' Theory and Gossen's Entwickelung der Gesetze des menschlichen Verkehrs (Berlin, Prager, 1854) they were considered the foremost exponents of the marginal utility concept. See Hutchison, Review, p. 14; also Schumpeter, History, pp. 483-486.

[17] There is a remarkable resemblance between the writings of Jevons on the one hand and those of both Senior and Gossen on the other. However, Jevons arrived at his theory of value quite independently and did not know of these works until after the first edition of the Theory. See Jevons, ibid., pp. xxxii-xxxix.

von Thünen, MacLeod, Cliffe Leslie and Ingram.[18]

It is a credit to Jevons' genius that he was able to gather up the scattered raw materials which these writers had left behind and to employ them in the erection of his own theoretical structure. At the same time we must acknowledge the fact that several of his building materials were heuristically imperfect and should have been discarded at the outset. Like Senior before him, Jevons made only a slight attempt at analyzing the process of individual valuation and the subjective factors which influence it.[19] He wrongly identified marginal utility with marginal demand and market price, and actually had very little to say about pricing at all.[20] Furthermore, he treated each individual's utility as a function of a single commodity, rather than all the commodities involved in a person's consumption habits; this too was a characteristic error of the early utilitarians.[21]

The following analysis will attempt to uncover

---

[18]The writings of other economists to whom Jevons was indebted are listed in Appendix V to the Theory. See ibid., pp. 322-342.

[19]See H. J. Davenport, Value and Distribution, Chicago, University of Chicago Press, 1908, p. 334.

[20]Ibid., p. 336.

[21]See Hutchison, History, pp. 114-115. Edgeworth corrected this error by relating utility to a consumer's entire consumption or Versorgungslage. For further information the reader is referred to F. Y. Edgeworth, Mathematical Psychics, London, C. Kegan Paul & Co., 1881, pp. 31-42.

critical flaws such as these. Some of the major questions to be posed are: Given a distrust of the older classical economics, and the rudimentary tools of marginal analysis, how did Jevons trace the outline of a new theory based upon a union of mathematics and English utilitarianism? Does this new organon represent an improvement over the classical position on value? And finally, are the criticisms which Marshall directs at Jevonian theory consistent with twentieth century analysis, or is Marshall guilty of Ricardian "wrong-headedness," as Schumpeter would have us believe?

The reader will note that this paper is initially divided into two parts: Part I is devoted to a synopsis of Jevonian marginal utility analysis and has been drawn largely from Jevons' Theory (5th Edition, 1957), which forms the core of his deduction; in Part II the author offers a presentation and evaluation of Marshallian criticisms of the Theory.

Part I

Chapter II, "The Utilitarian Framework," treats Jevons' statements on the measurement and dimensions of utility, together with his formulation of the "laws of utility."

Chapter III, "A Preface to Distribution Theory: Consumer Allocation," demonstrates how Jevons applied the marginal utility apparatus to several simple allocation

problems.

Chapter IV, "Marginal Utility and the Exchange Mechanism," furnishes an outline of Jevons' theory of exchange. Important points to be touched on include: (1) the "law of indifference"; (2) "trading bodies"; and (3) the derivation of exchange equations. An attempt is made at tracing out the process whereby Jevons extended the barter concept to the more complex cases of exchange, involving many commodities and trading bodies.

Part II

Chapter V, "The Marshallian Compromise," diagnoses the criticisms which Marshall directed at Jevons' theoretical work. Emphasis is placed on the following critical points: (1) Are "real costs" an implicit assumption in Jevons' Theory? (2) Are Marshall's criticisms of Jevons' "poor technique" valid, or do they obscure the fact that Jevons' real contribution lies in his delineation of marginal utility, not in the method by which this delineation was accomplished? (3) Is Marshall justified in rebuking Jevons for his so-called "hedonics"? and finally (4) Does Jevons really fail to comprehend the role of time periods in economic analysis as Marshall implies? Each of these points will be treated in turn.

In conclusion, then, we will be concerned with appraising Jevons' contributions to the area of pure theory

both in terms of Marshall's comments[22] and those of contemporary critics, including Schumpeter,[23] Hutchison, Roll and Stigler.

One wonders why Jevons was better known during his lifetime for his statistical investigations into the business cycle and for various papers on currency and finance,[24] rather than for his eminently more important theoretical writings. The reasons for this can be traced to two sources. The first lies in the nature of Jevons himself:

> ...Jevons left hardly any personal pupils, a fact that was in turn due not only to lack of opportunity (he never taught in a strategic position) but also to his amiable modesty or lack of assertiveness....But it is also true that his work in economic theory lacks finish. His performance was not up to his vision. Brilliant conceptions and profound insights... were never properly worked out; they were stated as aperçus and so intermingled with old stuff as to look almost superficial.[25]

---

[22]Most of Marshall's criticisms have been drawn from his Principles of Economics (1890) and a posthumously published work entitled The Memorials of Alfred Marshall (1925).

[23]The reader should be aware of the fact that Schumpeter looked more favorably upon Jevons and the Austrians than many other contemporary critics; quite often his criticisms of Marshall reflect this bias, as we shall see in Part II.

[24]See W. S. Jevons, Investigations in Currency and Finance, London, MacMillan & Co., 1884; also his Money and the Mechanism of Exchange, London, H. S. King & Co., 1878; and Principles of Economics, London, MacMillan & Co., 1905.

[25]Schumpeter, History, p. 826.

But there is another, perhaps more significant reason: Jevons, as an "originator" in the best sense of the word, felt impelled to break with a tradition which had become the life blood of English economists. Marshall, in the role of a national leader,[26] was obliged to defend this tradition, to protect it from its detractors, and to dispel forever the "mists of ephemeral criticism."[27] It was inevitable that the sparks of a "Jevonian revolution" would be stamped out before they had time to kindle the fires of a new English school.

To test the mettle of Jevons' thesis and Marshall's antithesis the following is directed.

---

[26]See ibid., p. 840.

[27]See ibid., pp. 920-921.

# PART I

## JEVONIAN VALUE THEORY

In the last few months I have fortunately struck out what I have no doubt is the true Theory of Economy, so thorough-going and consistent, that I cannot now read other books on the subject without indignation.

William Stanley Jevons

## CHAPTER II

### The Utilitarian Framework

The genesis of Jevonian value theory can be traced to 1862 and the presentation of Jevons' "Brief Account of a General Mathematical Theory of Political Economy" before the British Association at Cambridge. It was Jevons' hope that the paper, however reminiscent of earlier writings by Whately, Senior, and Lloyd, might effect a reappraisal of Ricardian doctrine, which for more than a half-century had served as the sine qua non of economic science. To his disappointment the paper failed to attract even moderate attention. An exclusive reliance upon hedonistic ideas to explain value, a proclivity to dress economics in the "heavy" armor of the calculus, a disavowal of the popular cost-of-production principles of Ricardo and Mill--these were the ingredients of a pastry which members of the Association did not even care to sample:

> A true theory of economy can only be attained by going back to the great springs of human action--the feelings of pleasure and pain.[1]

---

[1] Jevons, Theory, p. 304. J. A. La Nauze has established February 19, 1860 as the date Jevons discovered the significance of marginal utility (See R. S. Howey, The Rise of the Marginal Utility School, 1870-1889, Lawrence,

As if to acknowledge its indifference, the Association dismissed the paper perfunctorily and never offered to publish it.

Publication of Jevons' aperçu was deferred until 1866, when it was printed in extenso in the Journal of the Statistical Society of London.[2] Yet this proved no more fruitful than had the 1862 presentation and, largely as a result of critical indifference, Jevons temporarily put pure theory to one side and resumed his statistical investigations into commercial crises.

When in 1868 (and again in 1870) Fleming Jenkin published several articles[3] bearing close similarity to Jevons' original abstract on value, Jevons recognized the need for the immediate publication of a more complete statement of his principles of economics. During the hurried months between December, 1870, and August, 1871, he addressed his energies solely to this task, quite frequently to the detriment of his health. The final product of his labors, the Theory of Political Economy, was accepted for publication by the MacMillan Company in September, 1871.

---

Kansas, University of Kansas Press, 1960, pp. 1, 225).

[2]W. S. Jevons, "Brief Account of a General Mathematical Theory of Political Economy," Journal of the Statistical Society of London, XXIX, 1866, pp. 282-287.

[3]See Fleming Jenkin, The Graphic Representation of the Laws of Supply and Demand, and Other Essays on Political Economy, London, London School Reprints, 1931.

(1) The Measurement and Dimensions of Utility

Comparison of Jevons' Theory with his 1862 paper shows that all the original fibers remain, occasionally distended, frequently rewoven, always strengthened:

> ...Repeated reflection and inquiry have led me to the somewhat novel opinion, that value depends entirely upon utility,...that we have only to trace out carefully the natural laws of the variation of utility...in order to arrive at a satisfactory theory of exchange, of which the ordinary laws of supply and demand are a necessary consequence.[4]

Moreover,

> ...it is clear that Economics, if it is to be a science at all, must be a mathematical science ....simply because it deals with (variations in) quantities.[5]

Jevons well realizes that the feasibility of applying the calculus to human feelings might be questioned. The mind cannot add or subtract feelings, thereby effecting a balance between what is pleasurable and what is painful. Jevons denies, therefore, that utility is measurable in the cardinal sense:

> ...There is no unit of labour, or suffering, or enjoyment....I have granted that we can hardly form the conception of a unit of pleasure or pain, so that the numerical expression of quantities of feeling seems to be out of question.[6]

Yet the implication is made that while utility cannot be

---

[4] Jevons, Theory, pp. 1-2.

[5] Ibid., p. 3.

[6] Ibid., pp. 7, 12.

measured directly, it might be measured indirectly by its observable effects.[7] He also implies that, as more and more statistical data becomes generally available to economists, they eventually would be able to quantify utility. For the present, however, Jevons proposes that money-price will serve as a fairly accurate "measuring rod of utility."[8]

At least two major errors in Jevons' initial statements might be noted. In the first place he implies that the utility of every commodity to its consumer is some function of the quantity of that commodity alone, rather than of total consumption. In simple mathematical notation an individual's total utility could be written:

$$f(x_1) + g(x_2) + h(x_3) + \ldots,$$

where $x_1$, $x_2$, $x_3$, ... represent commodities. Walras and Menger were also guilty of this error. The more correct approach can be credited to Edgeworth, who established

---

[7]Stigler observes that Jevons was a good deal more skeptical of the measurability of utility in his first (1871) than in his second (1879) edition; e.g., in the 1879 edition he deletes the following: "I confess that it seems to me difficult even to imagine how such estimations (of utility) and summations can be made with any approach to accuracy. Greatly though I admire the clear and precise notions of Bentham, I know not where his numerical data are to be found" (ibid., p. 12). See Stigler, "The Development of Utility Theory," Journal of Political Economy, LVIII, 1950, p. 341.

[8]Jevons, Theory, p. 13. Jevons explains that he "never attempt(s) to estimate the whole pleasure gained by purchasing a commodity; the theory merely express(es) that, when a man has purchased enough, he would derive equal pleasure from the possession of a small quantity more as he would from the money price of it" (ibid.).

that an individual's utility is a function of all commodities involved in his consumption pattern.

A somewhat more subtle error of Jevons' is found in his statements on interpersonal utility comparison. He emphatically states that this is impossible:

> ...The reader will find...that there is never, in any single sentence, an attempt made to compare the amount of feeling in one mind with that in another....Every mind is...inscrutable to every other mind, and no common denominator of feeling seems possible.[9]

Yet several such comparisons are made later on in the Theory when he passes on to a discussion of exchange, as will be noted in Chapter IV. The conditions by which one feeling is said to be greater than another had been set down by Bentham as early as 1823; Jevons is unequivocal in his acceptance of them:

> ...To a person considered by himself, the value of a pleasure or pain, considered by itself, will be greater or less according to the four following circumstances:--
>
> (1) Its intensity.
> (2) Its duration.
> (3) Its certainty or uncertainty.
> (4) Its propinquity or remoteness.[10]

Strictly speaking, every feeling has two dimensions: intensity and duration. Whether pleasure or pain, a feeling must last for a certain period of time, and while it

---

[9] Ibid., p. 14. See also Jevons' letter ot Cairnes, January, 1872 (Reprinted in R. D. Black's "Jevons and Cairnes," Economica, XXVII, August, 1960, p. 228).

[10] Bentham, Fragment, p. 151.

lasts, it must be more or less intense. The anticipation of future events must also be considered as a source of satisfaction: the more developed a person becomes mentally and socially, the more important his anticipation of future feelings will be. Yet the uncertainty of future events will tend to cause even the most civilized individual to discount them.

Given these circumstances of feeling, everyone will act

> ...to satisfy (his) wants to the utmost with the least effort--to procure the greatest amount of what is desirable at the expense of the least that is undesirable.[11]

All human conduct will turn upon this innate desire to maximize positive feelings, or, what is the same thing, to minimize negative ones. This is the key to mathematical treatment of economic concepts: as a science of human behavior, economics differs from other social sciences in being a calculus of feeling, treating maximum and minimum values.

Bentham's schema can be extended to define certain terms relevant to the science of economics. Commodity is given to mean "any object, substance, action, or service, which can afford pleasure or ward off pain,"[12] and utility, the abstract ability which is possessed by a commodity to

---

[11] Jevons, Theory, p. 37.

[12] Ibid., pp. 37-38.

satisfy some desire. Jevons warns that utility is not a quality inherent in the nature of things, as the writings of Smith and other "Classicists" imply; rather, it is "a circumstance of things arising out of their relation to man's requirements."[13] To a thirsty man a quart of water has a very high degree of utility; yet, after drinking all the water he requires, a point of satiety is reached beyond which any additional water has zero utility. Consequently, "...utility is not proportional to commodity; the very same articles vary in utility according as we already possess more or less of the same article."[14]

Conversely, discommodity refers to those objects which possess the quality of causing inconvenience or harm; ashes and sewage are good examples. As to the abstract concept, that is, the opposite or negative of utility, Jevons uses the term disutility, a notion which, according to Schumpeter, he was the first to develop.[15]

(2) The Law of the Variation of Utility

Since utility is measured by the additions made to an

---

[13] Ibid., p. 43.

[14] Ibid., p. 44.

[15] See Schumpeter, History, p. 1057. However original the idea of "disutility" seems, it is, nonetheless, confusing at times. "Walras' treatment was more elegant--he introduced the marginal utility of leisure in complete symmetry to the theory of consumption--but not much more instructive" (Stigler, "Utility Theory," p. 321).

individual's happiness, and such happiness is given as variable over time, Jevons proposes that certain natural laws of the variation of utility can be formulated. The deduction of these principles and the application of them to distribution and exchange processes comprise the largest segment of Jevons' Theory.

The "Law of the Variation of Utility" is predicated upon the aforementioned fact that utility bears no proportional relationship to commodity:

> ...Utility may be treated as a quantity of two dimensions, one dimension consisting in the quantity of the commodity and the other in the intensity of the effect produced upon the consumer.[16]

Suppose a day's supply of food to be divided into ten equal parts.[17] The utility of the first two-tenths is infinitely great, since these portions are absolutely essential to supporting life. The third portion, while less necessary than the first two, is highly desired; the fourth, of less significance; and the fifth, a matter of indifference. As successive portions are consumed, the degree of utility

---

[16] Jevons, Theory, p. 47.

[17] The division of a commodity into ten portions is purely arbitrary: "If we had taken twenty or a hundred or more equal parts, the same general principle would hold true, namely, that each small portion would be less useful and necessary than the last. The law may be considered to hold true theoretically, however small the increments are made; and in this way we shall at last reach a figure which is undistinguishable from a continuous curve" (ibid., pp. 47-48). This qualification is important as far as aggregate consumption is concerned, which will vary by quantities which are infinitely small compared with the total consumption.

attaching to each tenth will diminish, each increment becoming less necessary than the previous one. Jevons does not pretend to be the originator of the concept of diminishing utility, and notes that earlier writers, such as Senior in his "Law of Variety" and Banfield with his "Law of the Subordination of Wants," had given the principle a less rigorous statement.[18]

The concept can easily be translated to a diagram.[19] Measuring quantity of commodity on the horizontal axis (OX), and the degree of utility, or intensity of effect upon the consumer, on the vertical axis (OY), Jevons was one of the first economists to illustrate this principle:

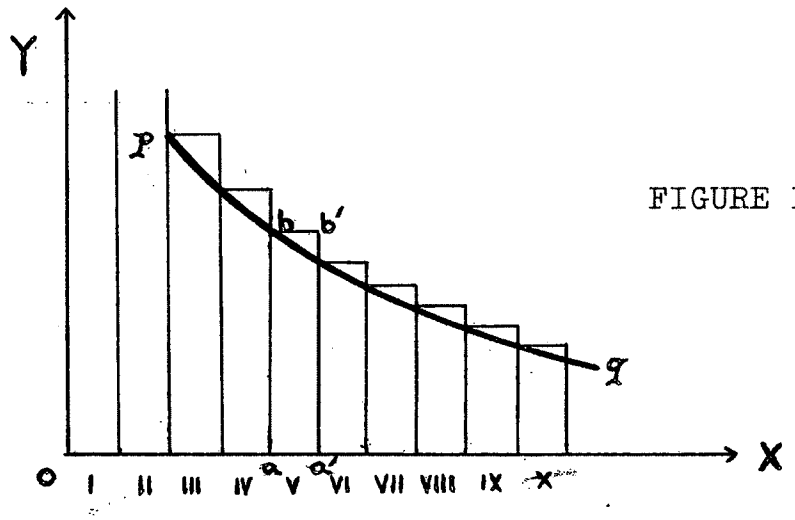



FIGURE I

---

[18]Schumpeter traces the concept to an even earlier writer, F. Galiani: "What separates Galiani from Jevons... is, first, that he lacked the concept of marginal utility... and, second, that he failed to apply his analysis to the problems of cost and distribution" (History, p. 301).

[19]To avoid repetition, the author has combined Jevons' Figures 3 and 4 (Theory, pp. 46, 49) in the above diagram,

> ...The law of the variation of the degree of utility of food may thus be represented by a continuous curve pbq...and the perpendicular height of each point of the curve above the line ox, represents the degree of utility of the commodity when a certain amount has been consumed.[20]

(3) The Law of the Variation of the Final Degree of Utility

A careful distinction must be made between the total utility arising from the consumption of any commodity and the degree of utility attaching to any given portion of it. In terms of Figure I these are shown by an area, aa'b'b, and a line, a'b', respectively. The total utility of the food individuals consume consists in maintaining life and is infinitely great; degree of utility, on the other hand, relates to changes in the quantity of commodity and total utility. In mathematical notation, if x is the quantity of commodity, or the independent variable, and u is the total utility arising from the consumption of x, or the dependent variable, then the degree of utility is

> ...the differential coefficient of u considered as a function of x, and will itself be another function of x....(or, symbolically) $\frac{\Delta u}{\Delta x}$....(and at the limit) $\frac{du}{dx}$.[21]

Degree of utility is important to Jevons only as regards

---

Figure I.

[20] Ibid., p. 48.

[21] Ibid., p. 51.

the last increment consumed, or the final degree of utility. This appears as the ratio of the increase in total utility to the increase in the quantity of commodity at the margin.[22] Failure to distinguish between total utility and final degree of utility had been the source of much confusion in economic thinking prior to Jevons' time.

Given this distinction,

> ...we may state as a general law, that the degree of utility varies with the quantity of commodity, and ultimately decreases as that quantity increases. No commodity can be named which we continue to desire with the same force, whatever be the quantity already in use or possession.[23]

Jevons accords credit to Jennings[24] for having first appreciated the importance of this "Law of the Variation of the Final Degree of Utility." He implies, as did Jennings before him, that the principle is founded on two parameters: in the first place, all wants are capable of satiety; and, secondly, as the satisfaction produced by one commodity diminishes, this good cannot be transferred to another use in order to achieve the greater satisfaction it initially produced, or, in other words, different commodities are not perfectly substitutable for one another in the satisfaction

---

[22]Although Jevons never used the term "marginal utility," this is what he is referring to in his discussion of the "final degree of utility." Schumpeter credits von Wieser with having invented the former phrase (Grenznutzen), History, p. 1055.

[23]Jevons, Theory, p. 53.

[24]See Richard Jennings, Natural Elements of Political Economy, London, Longmans, 1855.

of certain wants.[25]

(4) Notation of Utility

Expressing the dimensions of economic quantities is a task no less difficult than the measurement of other physical quantities; in many respects economic notation is even more troublesome, since the science must delineate certain quantities in terms of rates of change, as in the case of compound interest determination. Apropos of utilitarian measurement, the economist is confronted with the problem that the final degree of utility is an instantaneous state, quite independent of time. That is to say, time enters only as a negative factor. How then can we distinguish between instantaneous final degree of utility, on the one hand, and total utility, which is given as a function of both intensity and duration, on the other?

Jevons proposes that the final degree of utility will

---

[25] In this writer's opinion Jevons made a more important contribution to the literature when he extended his Law of the Variation of the Final Degree of Utility in his Principles of Economics. In this work the principle is placed in a truer context, the monetary economy; it becomes an assumption upon which the law of demand is founded: "When sugar is cheapened larger quantities in the whole will be demanded; this will arise from some who have already used it using more, from some who did not use any beginning to use it" (Principles, p. 57). Jevons also allows for the variability of income among various classes: "The general rule that as a commodity is cheapened its demand is increased...also depends upon the fact that the poorer classes of society are far more numerous than the richer, so that the cheapening brings it within a constantly expanding area of buyers" (ibid., p. 58).

ultimately depend upon the rate of supply of a given commodity. This rate can be denoted as $MT^{-1}$, where M is the absolute amount of commodity, and $T^{-1}$, negative time. If U, the degree of utility, is multiplied by $MT^{-1}$, the resulting notation, $MUT^{-1}$, will represent the real instantaneous state of feeling.

> ...The kind of quantity thus symbolized by $MUT^{-1}$ must be interpreted as meaning so much commodity producing a certain amount of pleasurable effect per unit of time.[26]

However, Jevons warns that this instantaneous state of feeling must not, as had been done by various members of the Oxford Utility School, be confused with the total quantity of utility:

> ...We must multiply this last symbol by T in order to obtain the dimensions of utility or quantity of pleasure produced. But in making this multiplication, $MUT^{-1}$ reduces to MU.... thus time eliminates itself and we arrive at a quantity of two dimensions.[27]

In this somewhat prolix attempt at clarifying previous statements on the dimensions of value, Jevons merely obscures an obvious fact: that the marginal utility of a commodity is the first derivative of the utility function.

---

[26] Jevons, Theory, p. 66.

[27] Ibid., pp. 66-67. Wicksteed takes Jevons to task for an unsatisfactory use of mathematical reasoning: "...if we say with Jevons that total utility has two dimensions, MU, we must, I think, add that one of these dimensions, U, is a ratio and not properly a dimension at all" (P. H. Wicksteed, "On Certain Passages in Jevons' Theory of Political Economy," Quarterly Journal of Economics, III, 1889, p. 303).

What promises to be a lesson in illumination becomes an exercise in confusion!

The history of economic theory holds many examples of writers who persisted in confusing marginal and total utility. Smith had spoken of both "exchange value" and "use value," the former arising out of the amount of labor embodied in the production of a commodity, and the latter, from a commodity's "intrinsic" worth. Mill slightly improved on Smith's theory by presenting "exchange value" as a relationship:

> Value is a relative term. The value of a thing means the quantity of some other thing, or of things in general, which it exchanges for.[28]

But what precisely is value relative to? Jevons asks. If value implies a relation, how can it be "some other thing"? Similarly, how can a commodity "possess" intrinsic value, as though there were an abstract "thing" which lies within it? To be sure, there are certain qualities inherent in gold and diamonds which will influence their value, but such intrinsic qualities are not synonymous with the term "value." Rather, this word "...merely expresses the circumstance of (a commodity's) exchanging in a certain ratio for some other substance."[29]

From the fragmentary definitions of his predecessors

---

[28] J. S. Mill, Principles of Political Economy, London, Longmans, 1936, p. 478.

[29] Jevons, Theory, p. 77.

Jevons discerns three distinct meanings which have traditionally been confused:

> (1) Value in use = total utility;
> (2) Esteem = final degree of utility;
> (3) Purchasing power = ratio of exchange.[30]

For economic theory to take on any sort of clarity in the future Jevons maintains that one must avoid use of the term "value" altogether,[31] and substitute these meanings in its stead. He claims the last term, "ratio of exchange," as his own invention, though Le Trosne and Condillac utilized a similar one (le rapport d'échange).

Employing his earlier notations, Jevons gives symbolic statement to the three terms. Smith's "value in use," or total utility, is the integral of $U \cdot dM$ and its dimensions are $MU$. "Esteem," or the degree of utility, has a single dimension, $U$. Finally, "purchasing power," the ratio of exchange, is given as unity, since it is without dimension, and is written as $M^0$.

---

[30] Ibid., p. 81.

[31] The reader will note that Jevons reverts to using the term in a later discussion. See "Footnotes to Exchange: The 'True' Origin of Value," p. 65.

CHAPTER III

## A Preface to Distribution Theory: Consumer Allocation

> ...In 1870 there was no theory of distribution. Most English economists after Smith devoted separate chapters to rent, wages, and profits, but without important exception such chapters were only descriptive of the returns to the three most important social classes of contemporary England.[1]

The Jevonian position on distribution has been the subject of much controversy since it was originally stated in the first edition of the Theory. Several authors, including Stigler[2] and Roll,[3] question the originality of Jevons' distribution theory, viewing it as little more than "warmed-over Classicism." At the other extreme Hutchison[4] and Davenport[5] regard it as a major contribution to the literature. This apparent redundancy is at once explained

---

[1] G. J. Stigler, Production and Distribution Theories, New York, MacMillan & Co., 1941, p. 2.

[2] Ibid., p. 14.

[3] Eric Roll, A History of Economic Thought, Englewood Cliffs, N. J., Prentice-Hall, 1956, p. 383.

[4] Hutchison, Review, p. 42.

[5] Davenport, Value, p. 334.

when we realize that Jevons channeled his thoughts on distribution in two separate directions; consequently two distinct theories emerge, the first treating consumer allocation and the second, distributive shares. The discursiveness with which he attacks these problems is occasionally confusing to the reader and often obscures the originality of his thoughts. After treating consumer allocation, he passes directly to the exchange mechanism; further comment on distribution--as found in his discussion of labor and capital, for example--is therefore deferred until much later in the Theory.

Jevons would have done well to adopt Mill's method of presentation, treating production, distribution, and exchange in turn; here is a legacy which has been passed on to twentieth century analysis. For want of a better, more systematic method of presentation, the Theory sometimes suffers; in his readiness to criticize Mill at every opportunity Jevons seems blinded to the fact that Mill's Principles is far superior to the Theory as regards organization and topical treatment of economic ideas. Schumpeter suggests:

> ...Perhaps he owed more to Mill than he knew; he harbored a strong aversion to Mill's Principles, which he had to use in his teaching; but Mill's tergiversations, which are such excellent targets for rifle practice, may nevertheless have taught him many things.[6]

---

[6]Schumpeter, History, p. 826.

Despite these organizational difficulties, each of Jevons' approaches to distribution represents a somewhat original attempt to extend the marginal utility concept to the realm of distribution. The reader will note that only the first of these theories, as regards consumer allocation, is treated here. It is the author's opinion that Jevons' other statements on distribution represent a departure from the main themes of his value theory. Those who wish to investigate this subject further should see Appendix B.

As is the case with many of Jevons' concepts, his theory of consumer allocation has an archetype; in this instance it is Gossen's famous Second Law:

> ...In order to obtain the maximum sum of enjoyment, an individual who has a choice between a number of enjoyments, but insufficient time to procure all completely, is obliged, however much the absolute amount of individual enjoyments may differ, to procure all partially, even before he has completed the greatest of them. The relation between them must be such that, at the moment when they are discontinued, the amounts of all enjoyments are equal.[7]

In other words maximum utility will result from a uniform level of "want satisfaction." (So far as the author can ascertain, Jevons arrived at this concept quite independently, and did not discover Gossen's Entwicklung until eight years after the Theory was first published.)[8]

---

[7]Gossen, Entwickelung, p. 12.

[8]See Jevons, Theory, p. xxxv. Nor was he aware that Bernouilli had given voice to substantially the same idea in his "Specimen Theoriae" as early as 1738. (See

Reasoning from this principle, Jevons postulates two cases in which the concept will govern consumer allocation; the first treats the allocation of a single commodity among various uses; the second, the distribution of a commodity over time.

(1) Allocation Among Different Uses

By simple illustration Jevons shows how his "Law of the Variation of the Final Degree of Utility" can be extended to consumer allocation, given a single commodity and various means for employing it. He offers the case of an "isolated family" which possesses stock $s$ of a commodity capable of two distinct uses, $x_1$ and $y_1$; by definition $x_1 + y_1 = s$. Rational conduct will dictate that only the distribution which affords greatest utility will be chosen; at this point "...an increment of commodity (will) yield exactly as much utility in one use as in another."[9] $\Delta u_1$ and $\Delta u_2$ are given to denote the increments of utility which arise from the consumption of one more unit of commodity in uses $x_1$ and $x_2$, respectively. Thus, when the desired allocation is completed, $\Delta u_1$ ought to exactly equal $\Delta u_2$. To state it another way, at the point of maximum satisfaction the final degrees of utility in the two uses will be equal; at the limit the equation appears:

$$\frac{du}{dx}_1 = \frac{du}{dy}_2,$$

---

Schumpeter, History, p. 1055). [9]Jevons, Theory, p. 59.

where $x$, $y$ are equal to $x_1$, $y_1$, respectively. *Ceteris paribus*, this same reasoning will apply to all uses simultaneously, with the result that all commodities, if consumed "rationally," will generate maximum utility. Only under the rarest circumstances--as in the event of the scarcity of such an essential grain as barley--would this principle prove inoperative; in such instances the utility of the grain as food might far exceed its utility in other uses, even, Jevons muses, in the production of "alcoholic liquors."

(2) Allocation of Commodity Over Time

Jevons also directs his attention to a related allocation problem: given a single use for commodity, $s$, and $n$ days in which to consume it, how can this commodity be consumed over time with the greatest utility? In this instance the condition for utility maximization would be written

$$v_1 = v_2 = v_3 = \cdots = v_n ,$$

where $v_1$, $v_2$, etc. denote the final degrees of utility on each day's consumption. Yet, suppose that the consumer is uncertain as to how long the stock of commodity will last; how will uncertainty condition his allocation? Jevons proposes that his future pleasures or pains must be reduced in proportion to their want of certainty; letting $p_1$, $p_2$ ... $p_n$ denote the successively diminishing fractions of uncertainty--and again assuming consumer rationality--he obtains

the following maximization condition:

$$v_1 p_1 = v_2 p_2 = \cdots = v_n p_n .$$

> ...The general result is, that as the probability is less, the commodity assigned to each day is less, so that $v_1$, its final degree of utility, will be greater.[10]

Similarly, the allocation must also account for the varying influence of an event according to its "propinquity or remoteness." Individuals tend to place greater value on present feelings rather than future feelings which are absolutely certain to occur. The above maximization conditions must therefore be amended to include some "discounting factor." Jevons designates these fractions of discount by $q_1, q_2, \cdots q_n$ . The new maximization equation is written

$$v_1 p_1 q_1 = v_2 p_2 q_2 = \cdots = v_n p_n q_n .$$

> ...It will be an obvious consequence of these equations that less commodity will be assigned to future days in some proportion to the intervening time.[11]

This distribution problem is often faced by ships about to make a voyage of uncertain duration. Given the proper wind conditions, the vessel might make passage in as little as ten days; at worst the voyage will extend to as many as thirty days. Were the ship's supply of food to be divided into thirty equal parts, partial starvation would develop during the first ten days--all this to ward off later evils

---

[10] Ibid., p. 72.

[11] Ibid., p. 73.

which might not even take place. Yet, consumption of one-tenth of the food on each of the first ten days might even be worse, for complete starvation would certainly ensue on the eleventh day. The most beneficial distribution would have to account for two factors: (1) the law of variation of the degree of utility of food; and (2) the probability of each day between the tenth and thirtieth days becoming part of the voyage. Jevons suggests that

> ...the allotments to the first ten days should be equal. They should afterwards decrease according to some regular law; for, as the probability decreases, the final degree of utility should increase in inverse proportion.[12]

********

While there is much to be said for Jevons' initial statement on the distribution process, it is disconcerting to find that he treats distribution only as regards the satisfaction of consumers' wants. For this reason marginal utility appears important only with regard to consumer goods and services; virtually nothing is said of the utility attaching to the factor inputs used to create these goods and services. Schumpeter observes that others--Gossen and Walras are notable examples--were equally guilty of this error. Of all the Austrians only Menger

> ...went on to say that means of production--or, as he called them, 'goods of higher order'--come within the concept of economic goods by virtue of the fact that they also yield

---

[12] Ibid., p. 74.

> consumers' satisfaction, though only indirectly, through helping to produce things that do satisfy consumers' wants directly.[13]

Menger's generalization of this concept allows us to treat all factors of production as "incomplete" consumable goods; the principle of marginal utility can thus be extended to include all areas of production and distribution. More than anything Jevons' failure to apply this principle to production theory represents a technical defect in his writings. One could easily argue that were it not for the exigency of readying his Theory for the press in such a short time, he might have attended to this defect; there can be no doubt that he had the necessary marginal tools for handling this. Yet, to impute any more than this to Jevons does discredit to Menger and later writers who carried marginal analysis beyond the realm of consumption.

---

[13]Schumpeter, History, pp. 912-13.

## CHAPTER IV

### Marginal Utility and the Exchange Mechanism

> ...The first problem that Jevons...tackled by means of the marginal utility apparatus was the problem of barter. Like (his) 'classic' predecessors, (Jevons) realized the central position of exchange value although, also like these predecessors, (he) did not make it sufficiently clear ...that exchange value is but a special form of a universal coefficient of transformation on the derivation of which pivots the whole logic of economic phenomena.[1]

Many would argue that at no point in his Theory does Jevons venture more than a few inches outside the framework of a static barter economy. In this sense his visions were similar to those of his intellectual predecessors. Yet others would defend Jevons as both progenitor and prophet of modern economic theory. His treatment of exchange serves as a good instrument for testing the mettle of this argument, for it is here that the heuristic significance of Jevonian theory is most evident: Jevons' "law of indifference" was later translated into Edgeworth's famous "indifference-curve" approach to demand analysis;[2] Walras used Jevons' marginal utility concept as a device for

---

[1] Schumpeter, History, p. 911.

[2] Edgeworth, Mathematical Psychics, pp. 28-29.

constructing the first general equilibrium system;[3] even Marshall unwittingly drew on Jevons' utility theory, transforming it into a doctrine of real cost.[4] That these and other men used Jevons' discussion of the exchange mechanism as a critical springboard for their own deduction would indicate that, more than anything, the Theory of Political Economy was important heuristically.

Yet, the author does not mean to imply that Jevons' treatment of exchange is without its faults; to the contrary, his errors are often painfully obvious. In the first place Jevons lacks a general theory of the determination of prices.[5] At the outset he makes the wrong assumption that the principles which govern two-party, two-commodity barter can be applied equally as well to perfectly competitive exchange. In making this assumption he fails to understand the connection between subjective utility estimation and the formation of market prices. For the most part he regards market prices as already "given" and

---

[3]Leon Walras, Elements of Pure Economics (trans. William Jaffe), Homewood, Ill., Richard D. Irwin, 1954, pp. 153-164.

[4]Schumpeter, History, p. 1057.

[5]Robertson suggests that MacLeod, an economist who greatly influenced Jevons, actually surpassed Jevons in one respect: he went beyond value theory to consider the (more relevant) theory of price determination; it is surprising that Jevons, a man who relied so heavily on the writings of MacLeod, therefore, did not also venture into price theory. See R. M. Robertson, "Jevons and His Precursors," Econometrica, XIX, 1951, pp. 237-238.

is concerned with them only as they relate to marginal utility when an equilibrium has been reached.[6]

A second major error in Jevons' treatment of exchange is found in his misuse of mathematics. Admittedly, Jevons did not pretend to be a mathematician.[7] He viewed mathematics as an important device for clarifying economic concepts, as indeed it is. But his mathematical approach is often poorly handled. Schumpeter observes

> ...we must see (in Jevons' exchange theory) an embryonic theory of general equilibrium or, at all events, a particular form of the unifying principle that is at the bottom of any general-equilibrium system. (But the reason he) did not make it fully articulate (is because he) did not understand the meaning of a set of simultaneous equations...[8]

The third major error which Jevons commits is to be found in the preface to the Theory's chapter on exchange, in which he sets about defining certain pertinent economic terms: "markets," "trading bodies," and the "Law of Indifference."

His definition of the "market" is taken from Cournot and Cantillon:

> ...By a market I shall mean two or more persons dealing in two or more commodities, whose stocks

---

[6]See Roll, History, pp. 380-381; also, Hans Mayer, 'Der Erkenntniswert der funktionellen Preistheorien', Die Wirtschaftstheorie der Gegenwart, Berlin, Prager, 1932, pp. 181-182.

[7]See Jevons, Theory, pp. xiii-xiv.

[8]Schumpeter, History, p. 1057.

> of those commodities and intentions of exchanging are known to all. It is also essential that the ratio of exchange between any two persons should be known to all others. It is only so far as this community of knowledge extends that the market extends.[9]

Hence, by definition, he excludes imperfectly competitive forces from the market altogether. Whether or not Jevons intended to treat these forces at a later date, when he would have been less pressed for time, is a matter of conjecture; his deflection from pure theory to statistical investigations (circa. 1872) would indicate that he did not propose to extend his analysis to imperfect competition. At any event, the generalized case of market imperfections was never formulated.

One might take even greater issue with Jevons' definitions of the "Law of Indifference" and "trading bodies." The former assumes perfect homogeniety of all commodities; that is to say, any portion of one commodity may be used in place of any equal portion of another. Since, ceteris paribus, there can be no reason why persons would treat exactly similar things differently, all portions of a good must be exchanged at the same ratio in the same market simultaneously. In other words

> ...in the same open market, at any one moment, there cannot be two prices for the same kind of article...I propose to call this The Law of Indifference, meaning that, when two objects or commodities are subject to no important

---

[9]Jevons, Theory, pp. 85-86.

> difference as regards the purpose in view, they will either of them be taken instead of the other with perfect indifference to the purchaser.[10]

Once more Jevons is postulating the case of a perfectly competitive market. Taken in this context, the Law of Indifference is a useful analytical device. Yet, in a "real world" sense, the concept is virtually useless. Jevons' error is in his assumption that real world and competitive analysis were one and the same. Though he allows for such factors as defective credit of purchasers, their imperfect knowledge of the market, etc., these elements are considered to be extraneous to the market[11] and therefore are excluded from the analysis. Robinson observes that this was a common error of most writers of the period; these economists

> ...misled by the logical priority of perfect competition in their scheme, were somehow trapped into thinking that it must be of equal importance in the real world. When they found in the real world some phenomena...which is inconsistent with the assumptions of perfect competition, they were inclined to look for some complicated explanation of it, before the simple explanation occurred to them that the real world did not fulfill the assumptions of perfect competition.[12]

---

[10] Ibid., pp. 91-92. Jevons adds that, given these conditions: "...the ratio of exchange at any moment is that of $dy$ to $dx$, of an infinitely small quantity of one commodity to the infinitely small quantity of another which is given for it" (ibid., p. 93).

[11] Jevons proposes that such factors will produce "unnatural ratios of exchange" (ibid., p. 86).

[12] Joan Robinson, The Economics of Imperfect Competition, London, MacMillan, 1942, pp. 3-4.

This dichotomy was later realized by Edgeworth, Walras, Marshall, and Wicksell, who, with varying degrees of success, attempted to place Jevons' Law of Indifference in a truer perspective.

Economists are in general agreement that Jevons' definition of the "trading body" is, at best, clumsy.[13] It is sketched in the broadest of terms as "...any body either of buyers or sellers....either an individual or an aggregate of individuals..."[14] who trade in the open market. Farmers are a trading body when they sell their produce to millers; millers are one when they sell flour to bakers; and so forth. Jevons looked on this concept as a natural outgrowth of the Law of Indifference: each trading body is indifferent to choosing any single homogeneous commodity over an identical one with the result that a single price prevails. Each trading body has an effect on the market and, in turn, is affected by the market.

The present author cannot accept Jevons' trading body concept for at least two reasons. In the first place it completely obscures the distinction between barter and

---

[13]Young notes that if this concept "were taken in any literal sense the market could not be supposed to be competitive. With all the millers and all the bakers in England conceived rigidly as a single pair of traders, the 'law of indifference' could not be invoked, and the equation of exchange would not lead to a determinate ratio of exchange" ("Jevons' Theory," p. 586). See also Eckard, Economics, p. 21; Roll, History, pp. 381-382.

[14]Jevons, Theory, pp. 88-89.

competitive exchange. Are we to assume--as Jevons does time and again--that the very same conditions hold irrespective of the number of commodities or parties to exchange?[15] So the concept would imply. Or can we accept Jevons' inference that the conditions of equilibrium are satisfied by a single price?[16] Again, decidedly not. We must agree with Hutchison, who suggests that Jevons apparently

> ...intended to make one model cover both two-party and two-commodity barter, and a competitive market in a monetary economy. As a result neither case gets clearly formulated.[17]

Even more inadmissible is Jevons' implication that freely competitive exchange leads to some sort of social welfare maximization.[18] He infers that, since barter equilibrium gives rise to maximum satisfaction for two individuals, competitive equilibrium will fulfill this condition for any number of parties. This curious extention of Gossen's Second Law is a convenient device which Jevons uses to prove the universality of his exchange equations. But this device is both unnecessary and illogical.

It is to these and other points which we now turn.

---

[15]See Knut Wicksell, Über Wert, Kapital und Rente, London, London School of Economics Reprints, 1933, p. 48.

[16]See Roll, History, p. 381.

[17]Hutchison, Review, pp. 42-43.

[18]See Roll, History, p. 385.

(1) Simple Two-Party, Two-Commodity Exchange

> ...The keystone of the whole Theory of Exchange and of the principle problems of Economics, lies in this proposition--the ratio of exchange of any two commodities will be the reciprocal of the ratio of the final degrees of utility of the quantities of commodity available for consumption after the exchange is completed.[19]

As the most basic of all economic processes,[20] exchange can assume many forms, the simplest of which involves two trading bodies exchanging two commodities. Trading body A will exchange some portion of his commodity for a certain portion of trading body B's commodity, with the result that a gain in utility is effected for both parties. The terms of the exchange will be defined by the respective utility functions of the two traders; for example, if ten pounds of corn are given for one of beef, this implies that the owner of corn considers ten pounds of his commodity to be less useful than one of beef. The same holds true for the owner of beef, as regards his commodity. The process of exchange will be terminated only after no further gain can be realized:

> ...This point of equilibrium will be known by the criterion, that an infinitely small amount of commodity exchanged in addition, at the same rate, will bring neither gain nor loss of utility.[21]

---

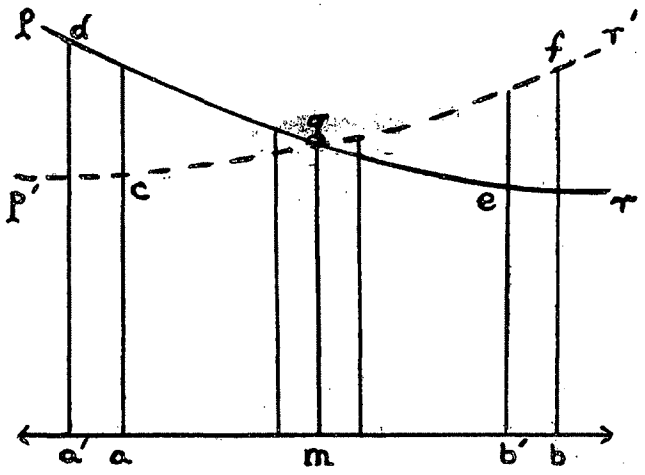
[19] Jevons, Theory, p. 95. See also Jevons' letter to Professor Walras (May 12, 1874) in Letters and Journal, pp. 302-304.

[20] See Jevons, Theory, p. 75.

[21] Ibid., p. 96.

Jevons utilizes a diagram to show the process by which this equilibrium is reached:

FIGURE II




In Figure II the curve pqr represents a small portion of the utility function for commodity A, while the broken curve, p'qr', is a similar curve for commodity B. This latter curve has been reversed and superimposed on the former one so that, while quantities of commodity A are measured from point a to point b, the quantities of commodity B are measured from b to a. Assuming units of both commodities to be represented by equal lengths, we see that line aa' represents an increase of commodity A, but a decrease of commodity B.

For the sake of simplicity Jevons proposes that the ratio of exchange is 1:1, so that, for example, by receiving commodity a'a a person gains utility ad, and loses utility a'c; hence, the net gain in utility is cd and the

individual will wish to extend the exchange. But how far? Jevons asks. Were this individual to continue it as far as point b', and he were still proceeding, through the next small exchange he would receive utility be. But in so doing he loses utility b'f, which implies a net loss of ef. Obviously he has gone too far, and should have discontinued the exchange at point q, the point of greatest advantage or "maximum utility." Beyond this equilibrium point net gain becomes net loss, with disutility as a result.

Jevons demonstrates how this first case of exchange might be translated into a simple "calculus."[22] He reverts to the notations which were used in his treatment of distribution; that is, $\Delta x$ represents a small increment of one commodity (corn) and $\Delta y$, a small increment of another commodity (beef) given for it. If both are homogeneous commodities, the Law of Indifference will prevail: $\Delta y$ will be to $\Delta x$ as $y$, the whole quantity of beef, is to $x$, the whole quantity of corn; or

$$\frac{\Delta y}{\Delta x} = \frac{y}{x} .$$

Jevons then incorrectly[23] assumes that by multiplying each

---

[22]"Jevons calls his mathematical processes calculus, but they correspond to the processes now taught in algebra" (Eckard, Economics, p. 18).

[23]Eckard questions Jevons' mathematical reasoning: "The expression $\frac{\Delta y}{\Delta x} = \frac{y}{x}$ discloses the relationship, adequately, but when he multiplies each side of the equation

side of the equation by $\Delta x$, he is able to obtain a similar relationship:

$$\Delta y = \frac{y}{x} \Delta x .$$

This equation would imply that $\Delta y$, the increment of beef, is $\frac{y}{x}$ times as great as $\Delta x$, the increment of corn; then, for their utilities to be equal, the degree of utility of beef must be $\frac{x}{y}$ times as great as the degree of utility of corn:

> ...Thus we arrive at the principle that the degrees of utility of commodities exchanged will be in the inverse proportion of the magnitudes of the increments exchanged.[24]

Exchange will alter both the holdings of commodity and the degrees of utility of the exchanging parties. Suppose that the total quantity of corn is $a$ and the total quantity of beef, $b$. In the process of exchange $x$ of corn is given for $y$ of beef; after it is terminated the corn owner holds $a - x$ of corn and $y$ of beef, while the beef-owner holds $x$ of corn and $b - y$ of beef. The utility functions may be designated as follows:

$\phi_1(a - x)$ = final degree of utility of corn to the corn-owner;

---

by $\Delta x$...he destroys the relationship between $\Delta x$ and $\Delta y$. This operation can be carried on, correctly, only with numbers, which is to say, abstract ideas. An increment of corn cannot be multiplied by a relationship between a quantity of corn and a quantity of beef!" (ibid., p. 23).

[24] Jevons, Theory, p. 100.

$\phi_2 x$ = final degree of utility of corn to the beef-owner;

$\psi_1 y$ = final degree of utility of beef to the corn-owner;

$\psi_2(b - y)$ = final degree of utility of beef to the beef-owner.

Therefore, the corn-owner will not be satisfied until the following equilibrium condition is fulfilled:

$$\phi_1(a - x) \cdot dx = \psi_1 y \cdot dy; \quad \text{or}$$

$$\frac{\phi_1(a - x)}{\psi_1 y} = \frac{dy}{dx}.$$

Assuming, as before, that $\frac{dy}{dx} = \frac{y}{x}$, Jevons substitutes for the second member of this equation, and obtains an equation which represents the equilibrium condition which faces the owner of corn:

$$\frac{\phi_1(a - x)}{\psi_1 y} = \frac{y}{x}.$$

Pari passu, what holds true for the corn-owner must likewise hold true for the beef-owner:

$$\psi_2(b - y) \cdot dy = \phi_2 x \cdot dx;$$

or, by substitution:

$$\frac{\phi_2 x}{\psi_2(b - y)} = \frac{y}{x}.$$

> ...We arrive, then, at the conclusion, that whenever two commodities are exchanged for each other, and more or less can be given or received in infinitely small quantities, the quantities

exchanged satisfy two equations, which may be thus stated in a concise form--

$$\frac{\phi_1(a - y)}{\psi_1 y} = \frac{y}{x} = \frac{\phi_2 x}{\psi_2(b - y)}$$ .[25]

Jevons proposes that these two equations alone are sufficient to determine the results of all forms of perfectly competitive exchange: it is simply a matter of solving for the "unknowns," in this case, $x$ and $y$, the amounts supplied and demanded.[26] The important notion that conditions of exchange might be represented by simultaneous equations dates back as early as Mill:

> ...the idea of a ratio, as between demand and supply, is out of place, and has no concern in the matter: the proper mathematical analogy is that of an equation. Demand and supply, the quantity demanded and the quantity supplied, will be made equal. If unequal at any moment, competition equalizes them, and the manner in which this is done is by an adjustment of the value.[27]

Jevons credits Mill with having first noticed this, while

---

[25]Ibid. In his discussion of Jevons' final equations of exchange, Roll notes that "he nowhere explained how these collective marginal utilities were determined. In fact, what he was considering was a case of isolated exchange, in which it is now admitted that the actual ratio of exchange is indeterminate within certain limits. It was left to Walras and others to show the connection between marginal utility, demand, and price under competitive conditions" (Roll, History, p. 382).

[26]Stigler observes that "quite aside from the ambiguous concept of the trading body, this procedure was illicit on (Jevons') own view that utilities of different individuals are not comparable" (Stigler, "Utility Theory," p. 319).

[27]Mill, Principles, p. 448.

at the same time criticizing him for not having shown how, given a discrepancy between supply and demand, they are ultimately brought into equilibrium. Another flaw in Mill's reasoning is his implication that exchange conditions can be stated in a single equation, when, in actuality, at least two equations are requisite. The symbol x in Jevons' final exchange equations (above) represents quantity demanded on one side and quantity supplied on the other; and similarly for symbol y. If these two equations are simultaneously true, according to Jevons, then the x and y of one equation will equal those of the other and the equilibrium exchange condition will be met. "...The laws of supply and demand are thus a result of...the true theory of value or exchange."[28]

(2) Exchange Between Two Trading Bodies of Unequal Size

The implicit assumption that all trading bodies are of equal size is unnecessary; indeed, if exchange analysis is to approach real world conditions, this assumption must be discarded. So long as commodities are capable of infinite

---

[28]Jevons, Theory, p. 101. At this point Jevons appears to come closest to understanding the true relationship between supply and demand, and their equality in equilibrium. This implication is again made in Appendix V of the Theory when he admits having employed "intersecting (supply and demand) curves to illustrate the determination of the market price in...lectures at Owens College (1863)." (Ibid., p. 333) Why Jevons did not include such diagrams in the Theory cannot be ascertained. Recent correspondence with the British Museum indicates that the 1863 lectures have not been preserved (see Appendix C, p. 118).

subdivision, the basic principles of exchange remain the same:

> ...We may, firstly, express the conditions of a great market where vast quantities of some stock are available, so that any one small trader will not appreciably affect the ratio of exchange. This ratio is, then, approximately a fixed number, and each trader exchanges at that ratio just so much as suits him.[29]

Let A be the trading body possessing two very large stocks of commodities, a and b. The other trading body, C, holds a comparatively small quantity, c, of the second commodity and exchanges a portion of it, y--which is very small compared with b--for portion x of a--which is very small compared with a. When the exchange is completed A will hold quantities a - x and b + y; conversely, C will hold x and c - y. This is shown by the equation:

$$\frac{\phi_1(a - x)}{\psi_1(b + y)} = \frac{y}{x} = \frac{\phi_2 x}{\psi_2(c - y)}.$$

Assuming that a - x and b + y are roughly equivalent to a and b, respectively, it is possible to substitute the latter quantities and obtain the equation:

$$\frac{\phi_1 a}{\psi_1 b} = \frac{y}{x} = m,$$

where m is given as the existing ratio of exchange.

Under these circumstances the ratio of exchange is

---

[29] Ibid., p. 112. In terms of modern theory Jevons is postulating the case of an individual firm in a perfectly competitive industry.

determined by the conditions facing trading body A and is comparatively fixed. Therefore, only one unknown quantity, x, must be solved for: this quantity will be purchased by C in exchange for some portion of c. The actual amount of c which is given in exchange will be determined by the equation:[30]

$$\frac{\phi_1{}^a}{\psi_1{}^b} = \frac{\phi_2{}^x}{\psi_2{}^{(c - mx)}}.$$

An even simpler exchange equation can be given to represent most of a consumer's daily purchases. Generally speaking, an individual will desire so little of a given commodity, such as salt, that he will give up only a small percentage of his possessions in order to obtain it. Suppose that y (in the above equation) represents only a very small part of c. In this case $\psi_2{}^{(c - y)}$ will roughly approximate $\psi_2{}^c$, and a single equation can be used to express the conditions of exchange:

$$\frac{\phi_2{}^x}{\psi_2{}^c} = m,$$

or $$\phi_2{}^x = m \cdot \psi_2{}^c.$$

Trading body C's exchange pattern will follow this general law: he will purchase the commodity until its degree of

---

[30] "This equation represents the position of an individual consumer with regard to the aggregate trade of a large community, since he must buy at the current prices, which he cannot in an appreciable degree effect." Ibid., p. 113.

utility falls below that of the commodity which he must give up:

> ...in the above equation $m \cdot \psi_2 c$ represents the utility to him of a penny, which being an inconsiderable fraction of his possessions, is approximately invariable in utility, and he buys salt until $\phi_2 x$, which is approximately the utility of the next pound, is equal to, or it may be somewhat less than that of the penny.[31]

However the same reasoning cannot be applied to the case of those purchases which form a large part of an individual's total consumption. This is especially true in the case of a poor family which is about to purchase a significant commodity such as meat. Only after they have secured a sufficient quantity of meat will the final degree of utility of this commodity begin to diminish:

> ...The more they buy, the lower the final degree of utility of the meat, and the higher the final degree of utility of something else; and thus these purchases will be the more narrowly limited.[32]

(3) Multi-Commodity Exchange Among Numerous Trading Bodies

An unfortunate assumption which Jevons makes is that the same principles which govern barter can be applied with equal facility to multi-commodity exchange among any number of competing parties:

---

[31] Ibid., p. 114. Jevons implies the demand for salt is less than perfectly elastic within a narrowly defined range below the market price.

[32] Ibid.

> ...Exactly the same principles hold true, however numerous and complicated may be the conditions. The main point to be remembered in tracing out the results of the theory is, that the same pair of commodities in the same market can have only one ratio of exchange...[33]

An explicit assumption behind this statement is that transport costs are non-existent. The more numerous the trading bodies and commodities become, the more equations will be needed to define the exchange process. The simplest example of this type of exchange would involve three commodities and three trading bodies, which face the following conditions:

> A possesses the stock a of cotton, and gives $x_1$ of it to B, $x_2$ to C.
> B possesses the stock b of silk, and gives $y_1$ of it to A, $y_2$ to C.
> C possesses the stock c of wool, and gives $z_1$ of it to A, $z_2$ to B.[34]

Solution of this exchange problem will involve six unknown quantities--$x_1$, $y_1$, $z_1$, $x_2$, $y_2$, $z_2$. The pattern of exchange is stated:

> A gives $x_1$ for $y_1$, and $x_2$ for $z_1$.
> B gives $y_1$ for $x_1$, and $y_2$ for $z_2$.
> C gives $z_1$ for $x_2$, and $z_2$ for $y_2$.[35]

Notation of the functions of utility is shown as:

$\phi_1, \psi_1, \chi_1$ : the respective functions of utility for A.

---

[33] Ibid., pp. 114-115.

[34] Ibid., p. 115.

[35] Ibid.

$\phi_2, \psi_2, \chi_2$ : the respective functions of utility for B.

$\phi_3, \psi_3, \chi_3$ : the respective functions of utility for C.[36]

When exchange is completed A will hold the quantity $a - x_1 - x_2$ of cotton and $y_1$ of silk; B, in turn, will possess $x_1$ of cotton and $b - y_1 - y_2$ of silk. The actual ratio of exchange is $y_1$ for $x_1$, and will be determined by a pair of equations:

$$\frac{\phi_1(a - x_1 - x_2)}{\psi_1 y_1} = \frac{y_1}{x_1} = \frac{\phi_2 x_1}{\psi_2(b - y_1 - y_2)} .$$

Other equations will apply to the exchanges between A and C:

$$\frac{\phi_1(a - x_1 - x_2)}{\chi_1 z_1} = \frac{z_1}{x_2} = \frac{\phi_3 x_2}{\chi_3(c - z_1 - z_2)} .$$

And similarly for interchange between B and C.

$$\frac{\psi_2(b - y_1 - y_2)}{\chi_2 z_2} = \frac{z_2}{y_2} = \frac{\psi_3 y_2}{\chi_3(c - z_1 - z_2)} .$$

Even the most complex exchanges may be decomposed into these simple cases, Jevons argues. It is merely a matter of considering every exchange as a process which gives rise to two equations, and solving for the quantities involved. Underlying this analysis is the assumption that all commodities are infinitely divisible; the condition of indivisibility is treated later in Jevons' Theory as one of several impediments to exchange.

---

[36] Ibid., p. 116.

(4) Competition in Exchange: 3 Trading Bodies

> ...One case of the Theory of Exchange is of considerable importance, and arises when two parties compete together in supplying a third party with a certain commodity.[37]

Imagine a situation in which trading body A, again in possession of quantity a of his commodity, is willing to exchange a portion of it for another commodity which both B and C possess; in amounts b and c, respectively. Suppose that A gives up $x_1$ of his stock, a, to B, and $x_2$ to C. In exchange A will receive $y_1$ of b (from B), and $y_2$ of c (from C). Assuming perfect homogeniety of commodity, we obtain a single ratio of exchange, based on the Law of Indifference:

$$\frac{y_1}{x_1} = \frac{y_2}{x_2} . \qquad (1)$$

So long as he receives the right commodity in the right amounts, trading body A will be indifferent as to its source. All he cares about is that he receives $y_1 + y_2$ in return for surrendering $x_1 + x_2$. Therefore:

$$\frac{y_1 + y_2}{x_1 + x_2} = \frac{y_1}{x_1} .$$

A will terminate the exchange only when the ratio of the final degrees of utility of the two commodities (a, and b,c) exactly equate the reciprocal of the ratio of exchange, as stated above:

---

[37] Ibid., p. 117.

$$\frac{\phi_1(a - x_1 - x_2)}{\psi_1(y_1 + y_2)} = \frac{y_1}{x_1} . \qquad (2)$$

The same principle must also hold for both B and C:

$$\frac{\phi_2 x_1}{\psi_2(b - y_1)} = \frac{y_1}{x_1}; \qquad (3)$$

$$\frac{\phi_3 x_2}{\psi_3(c - y_2)} = \frac{y_2}{x_2} . \qquad (4)$$

A solution of these four equations would involve determination of the values of the four unknowns, $x_1$, $x_2$, $y_1$, $y_2$.

(5) Impediments to Exchange: Transport Costs

The problem of formulating the principles of exchange is made no less difficult by the existence of certain elements which act to impede the exchange process. Included in these "impediments to exchange" are such factors as transport costs, charges of brokers, agents, packers, etc., and customs duties. All these charges and more will tend to reduce the advantages of commerce in this respect: such costs will usually be proportional to the quantity of commodity, and, if expressed in money, can be considered as some part of the commodity which must be subtracted if exchange is to occur; trading bodies will therefore have less commodity at their disposal. Thus, if trading body A gives trading body B commodity x, the latter receives only part of this commodity, mx; the actual quantity of commodity he receives will depend upon the fraction, m, which represents transport costs. B will terminate the exchange

only when

$$\frac{y}{mx} = \frac{\phi_2(mx)}{\psi_2(b - y)} .$$

Similarly, A will receive only ny in exchange, where n denotes the charges applied to y. In equilibrium

$$\frac{\phi_1(a - x)}{\psi_1(ny)} = \frac{ny}{x} .$$

The existence of transport charges will require that two ratios of exchange be established; the greater the discrepancy between these ratios, the less advantage there will be to exchange.

(6) Equivalence of Commodities

A special case of exchange will develop whenever two different commodities are applicable to the same uses: consumers of meats such as beef and mutton often appear almost indifferent to choosing one over another, with the result that their mutual ratio of exchange is practically invariable. Such commodities, to use Jevons' phrasing, are "more or less" reciprocally substitutable and the degree of substitutability will depend upon each good's final degree of utility. In 1871 the mutton-beef ratio was approximately 9 to 8, which would indicate that, while these commodities were nearly substitutable, people tended to place a slightly higher valuation on mutton:

> ...It follows that (if) the final degrees of utility of these meats are in this ratio, or that if $\phi$ x be the degree of utility of mutton

and $\psi y$ that of beef, we have

$$8 \cdot \phi x = 9 \cdot \psi y.^{38}$$

This then is the condition for the optimum allocation of consumer expenditures: in seeking to maximize his utility, a consumer will equate the ratio of the marginal utilities of the two goods with the ratio of their respective prices.[39]

Jevons was aware that under "rare" circumstances this equation would not fulfill the conditions for utility maximization:

> ...if mutton became comparatively scarce, there would probably be some persons willing to pay a higher price merely because it would then be considered a delicacy.[40]

The ultimate result of such conditions would be that the discrepancy between the prices of mutton and beef would

---

[38] Ibid., p. 135. Stigler observes that Jevons, in defining "substitutes" in terms of the constancy of the ratio of their marginal utilities, "was inconsistent, for he treated the marginal utility of $X_1$ as dependent only on the quantity of $X_1$ in his general theory, whereas $X_1$ and $X_2$ are "equivalent," the marginal utility of $X_1$ depends also on the quantity of $X_2$. One cannot define the usual relationships among the utilities of commodities with an additive utility function..." (Stigler, "Utility Theory," p.384).

[39] This differs from the modern optimum allocation position in an important respect: whereas Jevons speaks of equating price ratios with utility ratios, the present-day economist avoids the utility concept and is more concerned with the relative preferences of consumers. This would require that price ratios be equated with the marginal rate of substitution of commodities, i.e., the quantity of one good which would just compensate a consumer for the loss of a marginal unit of another good. See Appendix A, p. 103.

[40] Jevons, Theory, p. 135.

become even greater and, strictly speaking, these goods could no longer be regarded as near-perfect substitutes. However, he adds, this must be regarded as an extreme case. The more general rule is that

> ...the conditions of supply will have no effect upon the ratio of exchange; we must, in fact, treat beef and mutton as one commodity of two different strengths, just as gold at eighteen and gold at twenty carats are hardly considered as two but rather as one commodity.[41]

(7) Exchange Indeterminacy

Jevons' faith in the efficacy of his exchange equations is unyielding only to a point; in a somewhat discursive section of the Theory--entitled "Failure of the Equations of Exchange"--he admits that, owing to various parametric shifts in the conditions of exchange, it is possible to show several cases in which the exchange equations are inoperative. In the first place he postulates a case in which the utility functions of two trading bodies are so dissimilar as to preclude the existence of any trade at all:

---

[41] Ibid. Eckard observes that this statement is inconsistent with Jevons' principle of diminishing utility: "If the supply of beef fell off, people could only get more mutton by paying a higher price, unless the supply of mutton were perfectly elastic at the existing price. Hence, the conditions of supply do affect the ratios of exchange; for if mutton is produced under increasing cost, a higher price must be paid to call forth an additional quantity" (Economics, p. 24). Only under certain rarefied conditions would the ratios of exchange of two substitutes remain the same, although

> ...it may happen that the commodity possessed by A has a high degree of utility to A, and a low degree to B, and that vice versa B's commodity has a high degree of utility to B and less to A.[42]

Jevons maintains that, given these conditions, even though B receives a very small amount of A's commodity, the final degree of utility of this increment to him will be less than that attaching to his own commodity. Exchange will then benefit neither party and solution to the equations of exchange is indeterminate.

Conversely, under different circumstances A may exchange its entire commodity with B (and vice versa) and still the equations may fail:

> ...A may have so low a desire for consuming his own commodity, that the very last increment of it has less degree of utility to him than a small addition to the commodity received in exchange.[43]

In this instance each trading body desires the last increment of the good received more than it desires the last increment given; a determinate solution is again impossible.

Another parametric change occurs when Jevons drops his initial assumption that all commodities are subject to

---

Jevons fails to realize this.

Perhaps it is a minor point, but the author would also take issue with Jevons' analogy in the above statement: eighteen carat gold is easily converted into twenty carat gold, whereas such transformation is obviously impossible in the case of mutton and beef.

[42] Jevons, Theory, p. 119.

[43] Ibid.

infinite divisibility. This is obviously a more realistic assumption, especially as regards such discrete items of exchange as buildings and equipment. In the first place he postulates the case of two parties, each of which is in the possession of a single indivisible commodity, as, for instance, a book. They are confronted with the problem of exchanging the whole commodities or doing without exchange completely. In this instance it is total utility, rather than final degree of utility, which is significant. Designating total utilities in the following way

$u_1$ = the utility of A's book to A,
$u_2$ = " " " A's " to B,
$v_1$ = " " " B's " to A,
$v_2$ = " " " B's " to B,[44]

Jevons concludes that exchange will occur only if

(1) $v_1 > u_1$ and (2) $u_2 > v_2$.[45]

Should one or both of these conditions not be fulfilled, there would be no reason for exchange to take place.

More complex problems can arise when an indivisible commodity if offered for one which is perfectly divisible; a good example of this situation is found in the case of Russia's sale of Alaska, an indivisible "commodity," for American dollars, a divisible "commodity." In this

---

[44] Ibid., p. 121.

[45] Ibid.

instance the problem is determinate only within certain defined limits.

Another case could develop--and frequently does in retail trade--in which commodities are divisible, but insufficiently so. Commodities such as paper and wine are commonly produced in convenient, though discrete, units; to subdivide these units is to destroy the commodity completely. This problem would face a person buying ink. He must always weigh the total utility of a single bottle against the total utility of the shilling he gives for it, as illustrated in Figure III:[46]

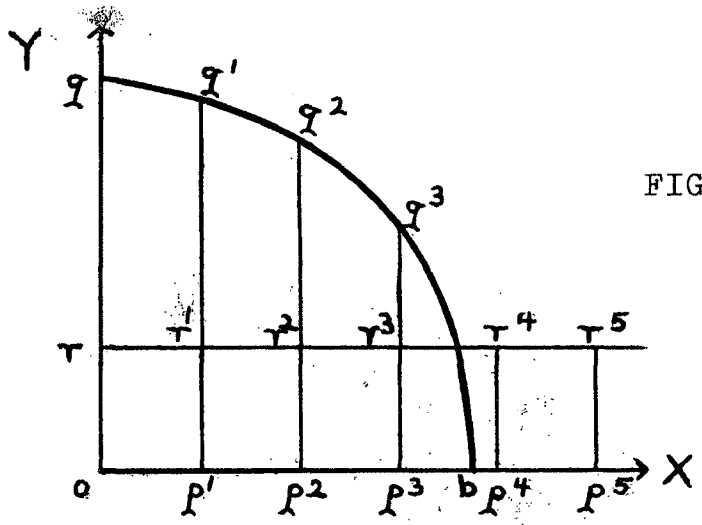



FIGURE III

In the above diagram the total utilities attaching to successive bottles of ink are shown by the spaces $\underline{Oq_1}$, $\underline{p_1q_2}$, $\underline{p_2q_3}$, etc., whereas total utilities of successive shillings are $\underline{Or_1}$, $\underline{p_1r_2}$, etc. Obviously, the individual will purchase no more than three bottles; were he to purchase the fourth

---

[46] Ibid., p. 126.

bottle the individual would make the disadvantageous sacrifice of shilling $p_3r_3r_4p_4$. So again the equations of exchange prove operative only within a reasonably well-defined area.

Once having formulated these cases, Jevons emphatically cautions the reader against accepting commodity indivisibility as the more general case. It is his contention that in practically all business dealings--especially in the area of international trade between great industrial nations--more or less commodity might be obtained in infinitely small quantities.

(8) Footnotes to Exchange: The "True" Origin of Value

As an architect of utilitarian economics Jevons is always quick to criticize what he viewed as imperfections in the basic framework of the science. However discursive his writings may appear at times, there can be no doubt that they reflect, above all, an attempt to smash the Classical labor theory idol.

Almost an entire century had passed since Adam Smith, the acknowledged progenitor of Classical economics, had spelled out the role of labor-cost in the determination of value. The wealth of nations, he proposed, is occasioned

> ...first, by the skill, dexterity, and judgment with which (a country's) labour is generally applied; and, secondly, by the proportion between the number of those who are employed in

useful labour, and that of those who are not so employed.[47]

Using Smith's theory as a point of departure, virtually every economist during the early part of the nineteenth century set about discovering the "true" origins of value. In his Principles of Political Economy (1820) Malthus defined value in terms of the "amount of stored and current labor" and utilized this definition to develop the concept of effective demand.[48] Torrens extended the labor theory to include capital, which he considered a type of "accumulated labor."[49] Similar statements were given by

---

[47]Adam Smith, An Inquiry into the Nature and Causes of the Wealth of Nations, New York, Modern Library, 1937, p. lvii.

[48]"It is obvious...that in the same place, and at the same time, the different quantities of day labour which different commodities can command, will be exactly in proportion to their relative values in exchange; and if any two of them will purchase the same quantity of labour of the same description, they will invariably exchange with each other" (Thomas Malthus, Principles of Political Economy, New York, A. M. Kelley, 1951, p. 94).

[49]"Thus, from the perpetually operating law of competition...it inevitably follows, that after the community divides itself into a class of capitalists and a class of labourers, the results obtained by the employment of equivalent capitals or equal quantities of accumulated labour, will be equal in exchangeable value" (Robert Torrens, Essay on the Production of Wealth, London, Longmans, 1821, p. 65). The concept of "accumulated labour" was not wholly original with Torrens and can be traced to Smith, who made implicit references to it in his discussion of productive vs. unproductive labor; see Smith, Wealth, pp. 194-195.

his contemporaries, James Mill[50] and McCulloch,[51] and do not need to be repeated here. Perhaps the most controversial variation on the labor theory belongs to Marx, whose "surplus value" concept forms the cornerstone of his famous Dialectic.[52] The important point is that each of these writers sought to explain value in terms of the labor involved in the production of a given commodity, rather than the subjective valuations which individuals place upon a commodity. This is not to imply, however, that these men wholly rejected the subjective-utility element. Ricardo and J. S. Mill, two of the foremost exponents of the labor theory, suggest that, while utility cannot be used as a measure of value, it is nevertheless essential to value:

> ...possessing utility, commodities derive their exchangeable value from two sources: from their scarcity, and from the quantity of labour required to obtain them.[53]

Ricardo implies that the first source is of somewhat lesser importance than the second: there are only a few

---

[50]See James Mill's Elements of Political Economy, London, Longmans, 1821, p. 94.

[51]See J. R. McCulloch's Principles of Political Economy, Edinburgh, A. & C. Black, 1849, pp. 372-373.

[52]"Surplus-value is produced by the employment of labour power. Capital buys the labour power and pays wages for it. By means of his work the labourer creates new value which does not belong to him, but to the capitalist" (Karl Marx, Capital, New York, Modern Library, 1932, p. 59).

[53]David Ricardo, Principles of Political Economy and Taxation, New York, MacMillan Company, 1921, p. 2.

Bell pp. 223

commodities

> ...the value of which is determined by their scarcity alone. No labour can increase the quantity of such goods (as rare statues and pictures), and therefore their value cannot be lowered by an increased supply.[54]

The more general case is that labor, both "current" and "stored up," will determine value.[55]

It was upon the basis of this apparent discrepancy[56] that Jevons built his case against Classical value theory: value cannot at one time be attributed to one cause and,

---

[54] Ibid.

[55] Ricardo, unlike many of his contemporaries, realized that labor itself was subject to variation. This required a modification of his earlier hypothesis that value was exactly proportional to the amount of labor embodied in the production of a given commodity. At first glance this modification appears tautological: one wonders how it is possible that labor, as the single most important cause of value, is itself amenable to unequal valuation. Eric Roll suggests that Ricardo's modification is a legitimate one: "Value remains determined by current and stored-up labour whether the latter belongs to the labourer or not. The only difference is that in the latter case the value of the product which is appropriated by the capitalist is divided into two parts, one which pays the wages of the labourer, the other which is the capitalist's profit" (Roll, History, pp. 178-179). See also George Stigler, "Ricardo and the 93% Labor Theory of Value," American Economic Review, XLVIII, June, 1958.

[56] There can be no doubt that Jevons' interpretation of Classical theory was often incorrect. At one point in his work, for example, he chides Ricardo for his implication that all labor is of uniform quality. (This was an incorrect interpretation of Ricardo, as noted earlier.) Jevons concludes that it is

> ...impossible to compare a priori the productive powers of a navvy, a carpenter, an iron puddler, a school master, and a barrister (Jevons,

at other times, to another:

> ...The mere fact that there are many things, such as rare ancient books...which have high values, and which are absolutely incapable of production now, disperses the notion that value depends upon labour. Even those things which are producible in any quantity by labour seldom exchange exactly at the corresponding values.[57]

By way of illustration Jevons cites the case of the (then) recently-completed Great Western Railway, the production of which required the services of a vast quantity of laborers. The value attaching to such a project cannot depend, however, upon the quantity of labor employed, but rather upon the number of persons who find the railway useful. Jevons felt that with the passage of time fewer and fewer people would find it useful and the total utility of the railway would eventually sink to zero. On the other hand an undertaking such as the Atlantic Cable could have a value far exceeding the amount of labor expended on it, at least for a time. It is therefore impossible to define value in terms of labor cost:

> ...<u>labour once spent has no influence on the future value of any article</u>: it is gone and

---

<u>Theory</u>, p. 166).

Given this assumption, Jevons proposes that the value of labor must ultimately be determined by the value of the produce, rather than vice versa. In other words, contrary to earlier statements by Ricardo and Mill, the value of labor is price-determined. (Eckard notes the parallelism between the above statement by Jevons and Cairnes' theory of reciprocal demand. See Eckard, Economics, p. 35.)

[57]Jevons, <u>Theory</u>, p. 163.

> lost forever. In commerce bygones are for ever bygones; and we are always starting clear at each moment, judging the values of things with a view of future utility.[58]

Yet the labor element must not be discounted entirely; though never the cause of value, labor is indirectly the determining factor in many instances:

> ...Value depends solely on the final degree of utility. How can we vary this degree of utility?--By having more or less of the commodity to consume. And how shall we get more or less of it?--By spending more or less labour in obtaining a supply.[59]

To summarize Jevons presents the famous value catena:

> ...Cost of production determines supply;
> Supply determines final degree of utility;
> Final degree of utility determines value.[60]

The end result is that value depends upon the conditions of demand in relation to a given supply; in this respect Jevons' position is antithetical to the position of those Classicists who treated changes in supply relative to a fixed demand.

As will be demonstrated in PART II, Jevons' value chain, with its simple causal relationships, is, like many of Jevons' statements on value and exchange, incorrect; but it is defective more in technique than in substance. It

---

[58] Ibid., p. 164.

[59] Ibid., p. 165.

[60] Ibid. It was this statement which prompted Keynes to comment that Jevons "chiselled in stone where Marshall knits in wool" (Keynes, Essays, p. 144).

remained for Alfred Marshall to pick away at the splinters of technical error which Jevons had unintentionally left in an unfinished building.

# PART II

## MARSHALLIAN CRITICISM

Jevons saw the kettle boil and cried out with the delighted voice of a child; Marshall too had seen the kettle boil and sat down silently to build an engine.

John Maynard Keynes

CHAPTER V

## The Marshallian Compromise

> ...There are few writers of modern times who have approached as near to the brilliant originality of Ricardo as Jevons has done. But he appears to have judged both Ricardo and Mill harshly, and to have attributed to them doctrines narrower and less scientific than those which they really held.[1]

Having already displayed the implements of Jevonian marginal utility analysis, we must now demonstrate the manner in which Jevons' major contemporary critic, Alfred Marshall, tested their mettle.[2] Of the many channels into which Marshall siphoned his assessments of the Jevonian theory, four were stressed and will concern us here: (1) Marshall's proposition that while English "classical" thought was defective in a number of aspects, its major

---

[1]Alfred Marshall, Principles of Economics, London, MacMillan & Co., 1898, p. 566.

[2]Marshall wins the title of Jevons' "major critic" by default: just as he had no major pupils, Jevons likewise had no major critics (i.e., reviewers), in the same sense as Mill was the great critic of Ricardo, Marshall of Mill, and Keynes of Marshall. The first of Marshall's criticisms of Jevons appeared as a review of the Theory of Political Economy in Academy, April 1, 1872. (See reprint in Memorials of Alfred Marshall, A. C. Pigou, ed., New York, Kelley and Millman, 1956, pp. 93-100.) Brief references to Jevons' work also were given in Marshall's Economics of Industry, London, MacMillan & Co., 1901,

tenets were fundamentally correct and heuristically significant; (2) his contention that Jevonian criticisms of classical analysis were to a large extent an outgrowth of a misreading of the older doctrine; (3) his remonstrance against Jevons for overemphasizing demand forces without regard for the equally significant elements of real cost and supply; and (4) the perfunctory commendation which he accorded to Jevons for having polished the utility side of the value coin.[3]

Schooled in the classical tradition, Marshall displayed a profound disregard for the iconoclasm of Jevons and the Austrian School in his early years. At the age of thirty he was asked by the editors of Academy to review Jevons' Theory:

> ...My youthful loyalty to (Ricardo) boiled over when I read Jevons' Theory....I have a vivid memory of the angry phrases which would force themselves into my draft (of the review), only to be cut out and then reappear in another form a little later on, and then to be cut out again.[4]

---

pp. 185, 242; Money Credit and Commerce, London, MacMillan & Co., 1924, pp. 20, 29, 279-280. The only "extended" criticism appears in Marshall's Principles, especially pp. 566-570.

[3]See Schumpeter, History, pp. 920-922.

[4]Memorials, p. 100. There were three other major reviews of the Theory: by John Elliot Cairnes in the Fortnightly Review, Simon Newcomb in the North American Review, and an anonymous writer in the Saturday Review. All differ in tone: only Cairnes' and Newcomb's could be called favorable. (See Howey, Marginal Utility School, pp. 62-63.)

Time and several reevaluations of Jevons' work tended to modify Marshall's feelings; a quarter of a century after his initial appraisal appeared Marshall wrote:

> ...I looked with great excitement for Jevons' Theory; but he gave me no help in my difficulties and I was vexed. I have since learnt to estimate him better. His manysidedness, his power of combining statistical with analytical investigations, his ever fresh honest sparkling individuality and suggestiveness impressed me gradually; and I reverence him now as among the very greatest of economists.[5]

This change in attitude on the part of Marshall cannot be called conciliatory; rather, it represents a recognition of the great theoretical similarity between the works of the two authors. Schumpeter writes:

> ...Marshall's theoretical structure, barring its technical superiority and various developments of detail, is fundamentally the same as that of Jevons, but...the rooms in this new house are unnecessarily cluttered up with Ricardian heirlooms, which receive emphasis quite out of proportion to their operational importance.[6]

This statement points to several significant questions which will form the substance of our diagnosis of Marshallian criticism: firstly, to what extent was Marshall's evaluation engendered by his disapproval of Jevonian technique, rather than by the actual substance of Jevons' Theory? And secondly was Marshall so enamoured with

---

[5] Memorials, p. 99. Despite his change in attitude, Marshall mentioned Jevons less than two dozen times in the later editions of Principles.

[6] Schumpeter, History, p. 837.

classicism and the "heirlooms" in its workshop that he was unable to discern the many technical errors which even the classicists had committed? Critical points to be touched on in our discussion of this latter question include: (1) the importance of "real cost" factors; (2) Jevons' use of mathematics; (3) the ethical implications of the hedonistic calculus; and (4) Jevons' incomprehension of the role of economic time periods.

A. Marshall and Jevonian Technique

In his Essays and Sketches in Biography John Maynard Keynes makes an analogy which neatly sums up the essential difference between Jevonian and Marshallian technique:[7]

> ...Jevons' Theory....(as) the first modern book on economics...(is) simple, lucid, unfaltering, chiselled in stone where Marshall knits in wool.[8]

The simile is appropriate. One receives the impression of two men very hard at work: Jevons, the hasty sculptor, tirelessly chipping away at bedrock, incognizant of either technique or detail; Marshall, the precise technician, stitching each new thread with faultless care, lest an unwanted flaw mar his fabric. As a master of technique, Marshall could not be expected to accept the apparent

---

[7]The controversy over economic technique greatly disturbed Marshall and prompted him to write: "There are nine or sixty ways of constructing tribal lays. / And every single one of them is right" (Quoted in Memorials, p. 318).

[8]Keynes, Essays, p. 144.

abandon with which Jevons had approached his Theory. (Marshall's own major work, Principles of Economics, was the product of more than twenty years' labor, during which time he also completed the Economics of Industry (1879) and two short monographs, The Pure Theory of Foreign Trade and The Pure Theory of Domestic Values). What concerns us here is whether or not Marshall's attention to technical detail colored his criticisms of the Theory.

Marshall was particularly impatient with Jevons' frequent utilization of causal chains to simplify relationships. It will be recalled that Jevons summarized his central position on value as follows:

> ...Cost of production determines supply;
> Supply determines final degree of utility;
> Final degree of utility determines value.[9]

Marshall writes:

> ...the greatest objection of all to (this) formal statement of his central doctrine is that it does not represent supply price, demand price and amount produced as mutually determining one another, but as determined one by another in a series. It is as though when three balls A, B, and C rest against one another in a bowl, instead of saying that the position of the three mutually determines one another under the action of gravity, he had said that A determines B, and B determines C. Some one else however with equal justice might say that C determines B and B determines A.[10]

Why not simply invert the order of the catena and express

---

[9]Jevons, Theory, p. 165.

[10]Marshall, Principles, p. 567.

the causal chain as follows?

> ...Utility determines the amount that has to be supplied,
> The amount that has to be supplied determines cost of production,
> Cost of production determines value.[11]

If we are to accept the catena in its original form, what prevents us, Marshall asks, from adopting this new form, imputing the cause of A to C? The critical point lies in the ambiguity of Jevons' third statement that "final degree of utility determines value." This seems to infer that market prices are arrived at merely through the interaction and ultimate balancing of one marginal utility against another. But Jevons avoids mention of an equally significant factor, the relative purchasing power of consumers, which he identifies with the "ratio of exchange."[12] This confuses cause with effect; price ratios are determined:

> ...not solely by the final degrees of (a good's) utility to them, but by these in conjunction with the amounts of purchasing power severally at their disposal. The exchange value of a thing is the same all over a market; but the final degrees of utility to which it corresponds are not equal at any two parts.[13]

Had he translated his law of diminishing (marginal) utility into the language of pricing, Jevons might have avoided

---

[11] Ibid. The implicit assumption behind this new chain is that cost of production ultimately determines that supply price which is requisite for keeping producers at their work.

[12] See Jevons, Theory, pp. 78-81.

[13] Marshall, Principles, p. 567.

this error; but the unfortunate confusion of marginal utility with market price and "marginal demand" (the term is Marshall's) remains. Davenport writes:

> ...The concept of marginal utility is, beyond question, of great significance in economic analysis, though...it is often most disastrously confused with marginal purchaser's price, that is, with relative marginal utility,--relative subjective worth, subjective value....But evidently it is only the latter concept that has any part or share in the term value as a market category, and expression of purchasing power. Value in this relational sense emerges only when utilities, as an individual category, have been, by different individuals, conceived relatively to other utilities to be displaced....In relative marginal utility, loss in terms of something else, the thought is carried over into the field of value.[14]

Marshall recognized this confusion and derived the concept of "marginal demand price," by which he meant the price which consumers are willing to pay for an additional unit of a given commodity.[15] By dropping the identity between marginal utility and marginal demand, Marshall is able to give clearer meaning to the concept of market equilibrium.

There can be little doubt that Marshall was the superior technician; he could not accept Jevons' oversimplification of causal relationships any more than he could subscribe to Jevons' fragmentary definitions of "trading bodies" and "the law of indifference." But he ignores a significant truth: Jevons' organon is important

---

[14]Davenport, Value, pp. 315-316.

[15]Marshall, Principles, p. 567.

not for the method in which it was presented, but for the fact that it was rendered at all. What Jevons says in the Theory is significant; how he says it is of lesser relevance:

> ...Jevons and the Austrians were held up to ridicule as people, who, like school children, had to be taught that 'when three balls...rest against one another in a bowl...the position of the three mutually determines one another under the action of gravity'....What Jevons and the Austrians really did was not the nonsense imputed to them in (this) passage but something very different; they discovered precisely that the position of the balls is to be accounted for by a single principle, gravitation in the case of mechanics, utility in the case of economics.[16]

This comment by Schumpeter reflects a particular school of thought which must, for want of a better term, be called "neo-Utilitarian." In contrast to Marshall--who, as we shall see, attempted to discount the viability of the marginal utility principle--Schumpeter proposes that this concept serves as the single most important factor in determining market behavior. To test this thesis the next section is directed.

B. Real Cost: An Implicit Assumption in Jevonian Analysis?

We generally credit Marshall with having first reconciled the utility and cost of production approaches to value, though the historiography of economics offers several examples of writers who anticipated this synthesis.

---

[16]Schumpeter, History, p. 922.

The earliest attempt can be traced to Galiani,[17] who, had he possessed the tools of marginal analysis, might have equalled Marshall's accomplishment. After Galiani we find the division of value theorists into two camps: the Classicists, who imputed the cause of value to cost of production and only paid lip service to subjective valuation (utility); and, secondly, the utilitarians, who reversed the "classical" argument and stressed subjective rather than cost elements. In either case there is an exaggeration of the importance of one factor over another: only when the former proves ineffective can the latter have any influence.[18]

Marshall found himself in the strange position of having to reconcile the two principles while, at the same time, attempting to defend the Classical case.[19] He proposed that "...the 'cost of production principle' and the 'final utility' principle are undoubtedly component parts of the one all-ruling law of supply and demand."[20] To impute the cause of value to a single factor is to tell only half a story. Each of the two principles which

---

[17]See ibid., pp. 300-302.

[18]Thus Mill writes: "...since cost of production here fails us we must resort to a law of value anterior to cost of production and more fundamental, the law of demand and supply" (Quoted in Marshall, Principles, p. 568).

[19]See "The Role of Time Periods," p. 93.

[20]Marshall, Principles, p. 569.

regulate value

> ...may be compared to one blade of a pair of scissors. When one blade is held still, and the cutting is effected by moving the other, we may say with careless brevity that the cutting is done by the second; but the statement is not one to be made formally, and defended deliberately.[21]

Standing behind demand is marginal utility, as expressed in the alternative prices which purchasers are willing to pay for various quantities of a given commodity; behind supply, marginal effort and sacrifice, which are reflected in supply prices:

> ...When demand and supply are in equilibrium, the amount of the commodity which is being produced in a unit of time may be called the equilibrium-amount, and the price at which it is being sold may be called the equilibrium-price....When demand and supply are in (this) stable equilibrium, if any accident should move the scale of production from its equilibrium position, there will be instantly brought into play forces tending to bring it back to that position; just as, if a stone hanging by a string is displaced from its equilibrium position, the force of gravity will at once tend to bring it back to its equilibrium position.[22]

In essence this statement must be accepted as correct: Marshall's partial equilibrium analysis compares favorably with what is taught in present-day courses in introductory economics. But if we are to go beyond this statement to the actual meanings of "demand price" and "real cost," we encounter some discrepancies. For instance, at times

---

[21] Ibid.

[22] Ibid., pp. 425-426.

Marshall seems as confused as Jevons had been over the relationship between marginal utility and market price. At one point he writes in criticism of Jevons:

> ...He had led many of his readers into a confusion...by speaking without qualification of the price of a thing as measuring its final utility, not only to the individual, which it can do, but also to a 'trading body' which it cannot do.[23]

But then he says:

> ...For each of (two men, one rich, one poor) the marginal utility is measured by sixpence; but this marginal utility is greater in the case of the poorer man than in that of the richer.[24]

His inference is that price may, after all, serve as a measure of marginal utility to an individual. But since Marshall elsewhere emphatically states that this is not the case, we must regard this statement as an unintentional error.

Marshall's handling of the other major determinant of value, "real costs," is a bit more awkward. He begins with the premise that there exists some relationship between the "real costs of production" and "expenses of production," but he seems uncertain as to the nature of this relationship. The former term refers to:

> ...the exertions of all the different kinds of labour that are directly or indirectly involved in making (a good); together with the abstinences

---

[23] Ibid., p. 176.

[24] Ibid., p. 170.

> or rather the waitings required for saving the capital used (in its production).[25]

"Expenses of production" on the other hand

> ...are the prices which have to be paid in order to call forth an adequate supply of the efforts and waitings that are required for making (a good); or, in other words, they are its supply price.[26]

Marshall's implication seems to be that real costs are those shared by persons who supply their factor inputs to the entrepreneur; while expenses of production are costs from the point of view of the entrepreneur himself.[27] Perhaps this is a valid distinction. But this still says nothing of the de facto relation between the two items. Are we to assume that the amount of sacrifice in effort and abstinence varies in direct proportion to the amount of the payments for productive services; or, furthermore, that there always exists an identity between the two amounts? Marshall never tells us. What is even more disturbing is his equation of expenses of production and supply price. If we are to accept his definition of the former--which includes only the minimum or "necessary" costs incurred by entrepreneurs--we must assume that entrepreneurs have nothing left in the form of profits; such an assumption is clearly unreasonable. We conclude then that this second identity is specious and that it will hold only when

---

[25] Ibid., p. 418.

[26] Ibid.

[27] See Davenport, Value, pp. 373-374.

profits are added to expense costs.

This brings us to an important question: is the concept of real cost implicit in the writings of Jevons? There is a good case which Schumpeter offers for the affirmative:[28]

> ...(Jevons) stood in no need of being told about the two blades of Marshall's pair of scissors. What (he) aimed at showing was that both blades consist of the same material--that both demand and supply (no matter whether the case is one of exchanging existing commodities or one of producing them) may be explained in terms of 'utility'.[29]

The reader will recall that Jevons introduces 'disutilities' on the same level as utilities: economics is the calculus of not only pleasure, but also of its converse, pain. In his treatment of equilibrium exchange[30] he proposes that beyond some point successive 'utils' can only be obtained at an increasingly greater sacrifice. At the fountainhead of disutility two forces are brought into play: exertion in the case of labor, abstinence in the case of capital. Regarding the first of these, Jevons writes:

> ...By far the most important instance of negative value is labour....Labour in the economic

---

[28] In defining the scope of this paper, the author has avoided Jevons' analysis of distributive shares, since this is another subject altogether. The following argument forms an exception to the prescribed rules, but this digression on labor and capital is necessary in order to corroborate Schumpeter's argument.

[29] Schumpeter, History, p. 922.

[30] See Jevons, Theory, pp. 95-98.

> sense of the term is essentially disutility, because it involves painful exertion; it is that which we give in production in order to obtain commodities. The labour given is painful to the giver....Thus the crossing-sweeper incurs tedious labour to ward off inconvenience and discomfort from the wayfarer.[31]

The disutility attaching to the acquisition of a rate of return on invested capital is also important, though less obvious. Capital is defined as:

> ...the aggregate of those commodities which are required for sustaining labourers of any kind... engaged in work....(It) allows us to expend labour in advance. Thus, to raise corn we need to turn over the surface of the soil. If we proceed straight to the work, and use the implements with which nature has furnished us--our fingers--we should spend an enormous amount of painful labour with very little result. It is far better, therefore, to spend the first part of our labour in making a spade...to assist the rest of our labour....(Thus) it is necessary to begin the spending of labour a long time before any goods can be finished.[32]

In other words only the crudest production process does not require some sort of abstinence; the more complicated a process becomes, the longer the period of abstinence. (There is an obvious parallel between this and Böhm-Bawerk's "roundabout production"[33] concept.) The rate of return on invested capital represents a payment for the disutility

---

[31] Jevons, Principles, p. 135.

[32] Jevons, Theory, pp. 223, 226-227. It is strange that Jevons, who was so disturbed by the classical emphasis on labor-cost, should frame his capital concept in these terms.

[33] See E. V. Böhm-Bawerk, The Positive Theory of Capital, London, MacMillan & Co., 1891, pp. 17-23.

incurred in abstaining from present consumption and is equal to "the rate of increase of the produce (as a function of time) divided by the whole of the produce."[34]

In short, Jevons uses the disutility concept as a vehicle for introducting cost elements into his fabric, but he does this only by implication. In contrasting his statements on disutility with Marshall's definition of "real costs" we find a similarity which neither author would probably have acknowledged. In both instances costs are measured in terms of 'sacrifice'; if we are to accept Schumpeter's argument, we find that the only important difference between the two theories is this: whereas Marshall's delineation of cost phenomena is articulate and precise, Jevons' is allusive and therefore more amenable to the criticisms of those who would discount the heuristic importance of the marginal utility concept.

C. Jevons' Use of Mathematics

The quest for a concise language for expressing their theories had led economists down three distinct paths by 1890: first were those writers--virtually all of the Mercantilists and Classicists fall in this category--who confined themselves to non-mathematical exposition;[35] next

---

[34]Stigler, Production, p. 27.

[35]"Numerical or algebraic formulations and numerical calculations had occurred of course in the earlier stages of economic analysis: there were the political

there were those who, mindful of the limitations of "literary economics," pioneered the graphic method: Dupuit (1844) and Jenkin (1871) were notable precursors of this approach; and finally in the writings of Bernouilli (1731), von Thünen (1826), Cournot (1838) and Jevons (1871) we see the growth of symbolic notation and mathematical economics. What we would call "modern analysis" is of course a blending of all three approaches; the synthesis is an accepted fact. But economists have come to recognize the value of all three approaches only within recent years. To anyone who has studied economic journals over the past seventy-five years the trend is obvious. It was not so apparent at the time Marshall wrote Principles in 1890. In contrast to Jevons' quasi-mathematical Theory, Principles stands as a monument to literary economics and the graphic method. Though he felt a great respect for mathematics, Marshall doubted its applicability to economic science. In a letter (1906) to Arthur Lyon Bowley he wrote:

> ...I know I had a growing feeling in the later years of my work at the subject that a good mathematical theorem dealing with economic hypotheses was very unlikely to be good economics: and I went more and more on the rules: (1) Use mathematics as a shorthand language rather than as an engine of enquiry. (2) Keep to them until you have done. (3) Translate into English. (4) Then illustrate by examples

---

arithmeticians, the physiocrats, and many isolated instances such as Brisco, Ceva...Condillac...(and to a limited extent) Ricardo...and Marx. But (this) does not constitute mathematical economics" (Schumpeter, History, pp. 954-955).

> that are important in real life. (5) Burn the mathematics. (6) If you can't succeed in (4) burn (3).[36]

More than anything Marshall feared that economists would become so enamoured with mathematics that they would leave off studying real world conditions and devote their energies to the construction of "economic toys."[37] This is the key to his indictment against Jevons' mathematics:

> ...Professor Jevons has expressed almost all of his reasonings in the English language, but he has also expressed almost all of them in the mathematical....We owe several valuable suggestions to the many investigations in which skilled mathematicians...have applied their favourite method to the treatment of economical problems. But all that has been important in their reasonings and results has, with scarcely an exception, been capable of being described in ordinary language....(The Theory) would be improved if the mathematics were omitted, but the diagrams retained.[38]

A skilled mathematician in his own right, Marshall was appalled by Jevons' not infrequent misuse of mathematical reasoning. In his review of the Theory Marshall writes:

> ...(Jevons) has not...fully availed himself of the accuracy which he might have derived from the use of (mathematics)....(In the first place) he does not always point out what are the variables as a function of which his quantities are expressed. It is often necessary to understand independently the whole of his reasoning, in order to know whether he means his differential co-efficients to be total or partial; and in

---

[36] Quoted in A. C. Pigou, Alfred Marshall and Current Thought, London, MacMillan & Co., 1953, pp. 8-9.

[37] See ibid., pp. 9-12.

[38] Memorials, pp. 97-99.

> several cases he seems almost to have himself forgotten that they are total.[39]

Marshall also questions Jevons' repeated ambivalence in symbolic notation, as manifested in the latter's shifting from differentiation to the delta process and back to differentiation.[40] To carry his point to an extreme, Marshall makes an attempt at integrating a number of Jevons' derivatives:

> ...A point on a locus may be determined by an equation with a differential co-efficient in it. (But) if we integrate (Jevons') equation, we get, not this locus, but some other intersecting it at the point to be determined.[41]

These errors might seem minor compared with Jevons' positive contributions, but their presence in a work of such importance was, in Marshall's estimation, clearly inadmissible. But here is where Marshall's criticism of Jevons' mathematics ends: he did not--as he might have done quite justifiably--pursue the point. And it is in this area that Jevons was most open to attack.

D. Ethical Implications of Utilitarian Economics: "Hedonics"

Of the many possible explanations for Marshall's distrust of the mathematical approach, one stands out above the rest: Jevons and other utilitarian writers had used mathematics to cement an illegitimate union between economics

---

[39] Memorials, p. 98.

[40] See Jevons, Theory, pp. 95, 98, 130.

[41] Memorials, ibid.

and ethics. "Hedonics,"[42] the product of this union, was a subject which must be outside the province of economic theoreticians, since it precludes the making of unscientific "value judgments":

> ...It has however unfortunately happened that customary uses of economic terms have sometimes suggested the belief that economists are adherents of the philosophical system of Hedonism or Utilitarianism. For, while they have generally taken for granted that the greatest pleasures are those which come with the endeavor to do one's duty, they have spoken of 'pleasures' and 'pains' as supplying the motives to all action.[43]

Jevons apparently thought he had absolved himself from the fallacy of hedonics when he wrote:

> ...I have never felt that there is anything in (hedonism) to prevent our putting the widest and highest interpretation upon the terms used.[44]

But in this regard his achievement was not up to his intentions: he walks a fine line between ethics and economics and often fails to distinguish between "desires" to maximize utility and "motives" behind these desires. Marshall writes:

> ...It is true that this large use of 'pain and pleasure' has sometimes served as a bridge by which to pass from individualistic Hedonism to a complete ethical creed....(but) it is clearly

---

[42]Marshall has inaccurately been accorded credit for having originated the phrase "hedonics." The author has traced its origin to a pamphlet by J. Grote entitled A Treatise on Moral Ideas. See the Oxford English Dictionary, Oxford, Clarendon Press, 1933, vol. V, pp. 189-190.

[43]Marshall, Principles, p. 77.

[44]Jevons, Theory, p. 23.

> not the part of economics to appear to take a side in ethical controversy.[45]

Yet all too frequently the accuser seems as guilty as the accused. Marshall's first edition of Principles bears most of the marks of utilitarian analysis, as can be seen in his habitual use of such hedonistic catch-words as "maximum pleasure," "minimum pain," and the phrase "terminal-value-in-use" (which corresponds to Jevons' "final degree of utility").[46] Subsequent editions were revised to weed out these utilitarian tracings--he replaced Jevons' "pain" and "pleasure" with the equally generic "dissatisfaction" and "satisfaction"--but the fact remains that Marshall was never quite able to free himself completely from the yoke of hedonics: "satisfaction," like Jevons' "pleasure," is to be pursued not for its own sake, but for the purpose of social betterment.[47] In principle Marshall had trouble accepting the unscientific alliance of economics and ethics; in practice he found it difficult to reject the role of social reformer. Pigou writes:

> ...economics for him was a handmaid to ethics, not an end in itself, but a means to a further end: an instrument, by the perfecting of which

---

[45]Marshall, Principles, p. 78.

[46]See Howey, Marginal Utility School, pp. 84-85; also P. T. Homan, Contemporary Economic Thought, New York, Harper, 1928, p. 270.

[47]See Memorials, p. 84.

> it might be possible to better the conditions of human life.[48]

E. The Role of Time Periods

Of no less concern to Marshall was Jevons' apparent incomprehension of the role of time periods and the effect which changes over time have on price; even Ricardo had been guilty of this error:

> ...The carelessness of Ricardo with regard to the element of Time has been imitated by his critics, and has thus been a source of twofold misunderstanding. For they attempt to disprove doctrines as to the ultimate tendencies, the causes of causes, the causae causantes, of the relations between cost of production and value, by means of arguments based on the causes of temporary changes, and short-period fluctuations of value.[49]

To an extent Marshall's statement is correct, for Jevons had virtually nothing to say about short- and long-run pricing:[50] the exchange rates which he arrives at always express marginal utility in relation to a fixed supply;

---

[48] Ibid., p. 93. See also 1883 lecture by Marshall, quoted in ibid., p. 83.

[49] Marshall, Principles, pp. 569-570.

[50] A brief summary of Marshall's position on time periods might be apropos: he distinguishes four periods, each of which is characterized by a different set of forces. In the first or "market" period "'supply' is taken to mean the stock of the commodity in question which is on hand" (ibid., p. 451). In this instance supply is perfectly inelastic and an increase in amount demanded will be matched by a concomitant, equal increase in market price. There is no entry or exit of either firms or factors. In the "short-period" we allow for factor changes: "'supply' means broadly what can be produced for the price in question with the existing stock of plant...in a given time"

that is, he assumes all supply curves are perfectly inelastic. But Marshall goes to the opposite extreme when he says:

> ...Thus...as a general rule, the shorter the period which we are considering, the greater must be the share of our attention which is given to the influence of demand on value; and the longer the period, the more important will be the influence of cost-of-production on value.[51]

What he appears to forget is his own principle of the pair of scissors: the forces behind both demand and supply will determine price, regardless of the time period under consideration. The only distinction that would even partially justify his statement is that in the long-run firms must always cover out-of-pocket or marginal costs, whereas this does not hold in the market period. Schumpeter criticizes Marshall's reasoning by way of an analogy:

---

(ibid.). Owing to the relative inelasticity of supply, an increase in amount demanded will result in a price which is greater than the original equilibrium price, but less than the aforementioned market price. Entry or exit of firms is impossible. In the long-run "'supply means what can be produced by plant, which itself can be remuneratively produced and applied within the given time" (ibid.) Factors will enter in order for firms to achieve optimal efficiency; firms will also enter in order to take advantage of the greater profit opportunities. Those facing long-run losses will leave. Given the increased supply potential (and assuming increasing costs), the long-run supply curve will be relatively elastic and at its point of intersection with the demand curve a "normal price" will prevail. Marshall's fourth period, commonly referred to as the "long, long run," involves secular movements of normal price "caused by gradual growth of knowledge, of population and of capital, and the changing conditions of demand and supply from one generation to another" (ibid.).

[51] Ibid., p. 429.

> ...In its general import...(this statement) gives the wrong lead...it is wrong to say that foreign exchanges are determined by supply and demand in the case of paper currencies, and by the gold mechanism in the case of gold currencies; what should be said is that the factors behind supply and demand determine foreign exchange rates, in any case, but that in the case of gold currencies the gold mechanism will in general prevent departure from gold parities beyond the gold points.[52]

Marshall seems to imply that the marginal utility and cost-of-production principles are independent of one another; but we know from the foregoing that this was not his intention. The fact is that at the center of Marshall's system is the "fictional stationary state" in which cost-of-production would always govern value or "normal price":

> ...Each element of cost would be governed by 'natural laws', subject to some control from fixed custom. There would be no reflex influence of demand; no fundamental difference between the immediate and the later effects of economic causes.[53]

Jevons and other writers before him had implied that dynamic analysis was virtually impossible, given the existing state of economic science.[54] Marshall regarded this as an evasion of the issue and used the stationary state device to bridge the gap between statics and what he wrongly called "dynamics." In this mythical state the influences

---

[52]Schumpeter, History, pp. 921-922.

[53]Marshall, Principles, p. 440. Marshall treats this for what it is: a device which is conceptually valuable but of no practical importance in the real world. (See ibid.)

[54]See Jevons, Theory, pp. 93-94.

which exerted themselves on value in the aforementioned periods become irrelevant:

> ...The general conditions of production and consumption, of distribution and exchange remain motionless; but yet it is full of movement; for it is a mode of life.[55]

There is no need to consider the effect which changes in tastes, technology or factor availability have on price, for they are assumed constant over time. And since this is so, entrepreneurs will come to expect this constancy; thus price-expectations also are unimportant:

> ...In the stationary state then the plain rule would be the cost of production ('normal cost' faced by a 'representative firm') governs value ('normal price').[56]

We conclude from the above that Marshall's analysis of the time element shows a marked advance over Jevons' implicit treatment of time in the Theory. Indeed, Marshall had just cause for criticizing Jevons' preoccupation with the "market period" and for his complete ignorance of short- and long-run equilibria.

But both men approached the same problem from different angles and achieved different results. Taken in context, neither treatment is incorrect: if we confine

---

[55] Marshall, Principles, p. 439. This device is supplemented by the concept of the "representative firm," a quasi-"average" unit whose economies depended upon the aggregate production of the good it produced. See ibid., pp. 397, 422, 450, 514.

[56] Ibid., p. 440. See also J. R. Hicks, Value and Capital, Oxford, Clarendon Press, 1946, p. 117.

ourselves to the market period, we have no problem justifying Jevons' statement that final degree of utility (demand) determines value; conversely, if we make use of Marshall's "stationary state" model, we can defend his thesis that value is determined by costs of production. What we cannot accept, however, is the fact that Marshall bases his criticism of Jevonian statics on a quasi-dynamic model. He apparently ignores the fact that Jevons explicitly stated that he was working within a static framework in which the so-called "streams of trade" were assumed constant. Once this is realized, we are forced to conclude that Marshall's criticisms are really irrelevant.

## CHAPTER VI

## Conclusion

Whenever we probe below the surface of a body of economic ideas, we are certain to uncover a number of factors which belie the importance of these ideas. So it has been with our brief synopsis and evaluation of Jevonian value theory. In PART I, when we traced the circuitous path by which Jevons expanded on the marginal utility principle, we discovered that many of his thoughts were not wholly original, but had been inherited from the English hedonists, the French mathematical writers, and various members of the Oxford Utility School. And again in PART II, when we examined the Theory in terms of Marshallian criticisms and Schumpeterian counter-criticisms, we questioned the validity of Jevons' generalization of marginal utility, and pointed to several significant technical errors in Jevons' deduction. We must now determine the extent to which these factors detract from the Jevonian organon.

Jevons' single most important achievement was the statement of the principle of diminishing marginal utility. It was this device which permitted him to break free of the Classical, production-oriented economics and to focus

consideration on the consumption problem. We have noted several antecedents to Jevons' treatment of this problem--Gossen's "Second Law" and Senior's "Law of Variety" are perhaps the most obvious--but Jevons must be credited with having first portrayed the problem in a rigorous manner, using mathematical notation. He is the first English writer to define marginal utility as the first derivative of an individual's utility function. Once the significance of this achievement is realized, the many technical flaws in his theory--the vague definitions, the naive "causal chains," and even his obtuse extensions of the basic exchange equation[1]--become irrelevant. Why then do many current economic historians discount the importance of Jevons' accomplishment?

We offer three explanations. In the first place most writers tend to group Jevons with his contemporaries--the Austrians, and Walras, and Pareto. Consequently, his achievements are dwarfed by the collective contributions of all these utilitarian writers. Only the less positive aspects of Jevons' deduction remain: (1) his treatment of marginal utility as a function of only a single commodity; (2) his failure to develop a dynamic model based on flow- rather than stock-variables;[2] (3) his confusion of barter

---

[1]Jevons, Theory, p. 100.

[2]See ibid, pp. 93-94; also Hutchison, Review, p. 43.

and freely competitive exchange conditions; (4) his exclusive reliance upon market period analysis; and (5) his so-called "hedonics." As we are reminded by current writers, the progress of economic science has underscored the seriousness of Jevons' errors. We must agree. What we cannot accept, however, is the alacrity with which these same writers dismiss Jevons as being the first English economist to present the marginal utility device rigorously.

A second explanation for the lack of concern for Jevons' work is an historical one. As we noted in PART II, Marshall took great pains to discount the significance of the Theory. There is little doubt that he succeeded. Moreover, he also succeeded in presenting a theory which drew largely on the very concept for which he had criticized Jevons. We must credit him with having put value theory in its proper perspective, by uniting the cost-of-production and utility approaches. But we must not lose sight of the fact that he was the originator of neither approach, and that, as regards the latter, much of what Marshall said had already been phrased in a less eloquent way by Jevons.

The final reason which we offer for the current inattention to Jevons' Theory relates to the problems inherent in utilitarian measurement. The de facto value of the marginal utility device remains today as a subject of economic controversy. Most writers, following Marshall's line of

reasoning to a certain extent, would question the real significance of this device on the grounds that it presupposes interpersonal utility comparison, poses behavioral questions which fall within the province of psychology and sociology, and leads economists into the unscientific realm of value judgments. Others, aligning themselves with the "neo-Utilitarians," consider the device so important as to make these difficulties seem insignificant. We touched upon this problem in PART II's digression on "real cost," at which time the author left it open for debate. His reasons for doing so can be best explained in a paragraph from Jevons' last completed work:

> ...We must agree to differ, and though we are bound to argue fearlessly, it should be with the consciousness that there is room for wide and bona fide difference of opinion. We must consent to advance cautiously, step by step, feeling our way, adopting no foregone conclusions, trusting no single science, expecting no infallible guide....We must recognize the fact clearly that we have to deal with complex aggregates of people and institutions, which we cannot usually dissect and treat piece-meal. Tolerance, therefore, is indispensable.[3]

---

[3]Jevons, The State in Relation to Labour, p. 166.

# APPENDIX

It was during the year 1851...that I began to think that I could and ought to do more than others. A vague desire and determination grew upon me....I felt it to be almost presumptuous to pronounce to myself the hopes I held and the schemes I formed. Time alone could reveal whether they were empty or real; only when proved real could they be known to others.

William Stanley Jevons

## APPENDIX A

### Consumers' Behavior: The Transition to Twentieth-Century Analysis

From the foregoing the reader might receive the false impression that demand theory begins and ends with Jevons' formulation of the marginal utility approach. However, to assume that his theory is the correct one, or even the only one, is to ignore a very significant part of economic literature. In this appendix the author will attempt to outline the major developments in demand analysis which took place in the interim between Jevons' Theory (1871) and Hicks' Revision of Demand Theory (1956).

Although the distinction between various approaches to demand is not always obvious, we can generally describe five stages[1] in the evolution of demand theory. We begin, of course, with Jevons. What characterizes this first stage has already been set out and may be summarized as follows: (1) acceptance of the cardinal utility principle, whether explicitly (in the case of the Oxford Utility School) or implicitly (in the writings of Jevons, Marshall,

---

[1]See Stigler, "Utility Theory"; also Hutchison, Review, pp. 303-307.

and Walras); (2) incomprehension of complementary relationships between goods (Marshall is the exception); and (3) treatment of marginal utility as a function of a single good.[2]

We can broadly define the second stage in the development of demand theory in terms of two contributions by Edgeworth, and Auspitz and Lieben, respectively: (1) the treatment of marginal utility as a function of the quantities of all goods involved in a person's consumption pattern; and (2) the introduction of complementarity between goods. It must be noted that all three of these writers anticipate "indifference curve analysis," although their method of presentation is less rigorous than that of Vilfredo Pareto, who is properly classed in the next period. In Mathematical Psychics Edgeworth combines indifference curves--which, unlike their present familiar form, are drawn concave to the origin--and price-lines to determine a point of maximum consumer satisfaction.[3] Auspitz and Lieben's "constant-satisfaction curves" bear a close similarity to the Edgeworth curves, although there is evidence that they arrived at their analysis independently.[4]

The writings of Pareto and Fisher reflect what we

---

[2] Ibid., p. 303.

[3] See Edgeworth, Mathematical Psychics, pp. 21-22.

[4] See R. Auspitz and R. Lieben, Untersuchungen Über die Theorie des Preises, Leipzig, Duncker & Humblot, 1889.

shall call the third period in the evolution of demand theory. Realization of the limitations imposed by the cardinality problem led these men to a non-utility analysis of consumer behavior. The earlier indifference curve analysis provided a means for extruding "utility" from value theory as being measureable. That they did not wholly succeed in accomplishing this can be attributed to the fact that neither of these men was able to redefine the law of diminishing utility and complementarity in non-utility terms.[5]

A major development occurred in the fourth stage with the introduction of the concept of the "marginal rate of substitution." Although we generally credit Hicks with having conceived this device, its origin can be traced to the writings of two earlier economists who published during the second decade of this century. W. E. Johnson's article on "The Pure Theory of Utility Curves" (1913) outlines the general form of the concept, however less rigorously than his successors:

> ...This impossibility of measurement does not affect any economic problem. Neither does economics need to know the marginal (rate of) utility of a commodity. What is needed is a representation of the ratio of one marginal utility to another. In fact, this ratio is precisely represented by the <u>slope</u> at any point of the utility curve.[6]

---

[5]See Hutchison, <u>Review</u>, p. 303.

[6]W. E. Johnson, "The Pure Theory of Utility Curves," <u>Economic Journal</u>, 1913, p. 490.

What Johnson states implicitly, Slutsky expresses explicitly in his "Sulla Theoria del Bilancio del Consummatore."[7] In this highly mathematical article Slutsky completes the extrusion of utility, thereby anticipating much of the later deduction by Hicks and Allen. He rightly deserves credit for having rigorously distinguished between the substitution and income effects of price changes.[8] A rediscovery of his work by Hicks and Allen took place in 1934, their findings being published in "A Reconsideration of the Theory of Value."[9] But perhaps the most representative work of this period is Hicks' Value and Capital (1939). In essence this book is an attempt to redefine consumer behavior in non-utility terms, thereby bypassing the unnecessary cardinality assumptions of earlier writers. Two of the more

---

[7]E. Slutsky, "Sulla Theoria del Bilancio del Consummatore," Giornale degli Economisti, LI, 1915. See also R. G. D. Allen, "Professor Slutsky's Theory of Consumer's Choice," Review of Economic Studies, IV, 1936.

[8]A complete separation of these two effects is possible only when we use the indifference-curve approach to demand. If we assume the price of X to fall relative to the price of Y, the effects can be isolated as follows: the substitution effect (with real income temporarily held constant) is shown by a shift along the same indifference curve towards the X axis; conversely, the income effect represents the change in real income which is generated by the price change, and would be shown by a movement to a higher indifference curve. For further information see R. H. Leftwich, The Price System and Resource Allocation, New York, Rhinehart & Co., pp. 87-90; also Hicks, Value, pp. 32-34.

[9]J. R. Hicks and R. G. D. Allen, "A Reconsideration of the Theory of Value," Economica, III, 1934.

outstanding devices which Hicks uses to accomplish this are (1) the marginal rate of substitution, which replaces Jevons' marginal utility; and, following from this, (2) the principle of diminishing marginal rate of substitution, supplanting diminishing marginal utility. Each indifference curve will show a given scale of preferences which an individual consumer applies to two commodities. According to this approach consumers are unable to quantify the satisfactions derived from the consumption of a good. All that can be said is that each individual will prefer one scale of preferences to another, the former being represented by a higher curve on his "indifference map." We must assume that the two goods displayed on the map are substitutable, so that as relative prices change the substitution effect can be brought into play. At any single point along an indifference curve we can define its slope in terms of a ratio between the marginal utilities of the two goods. Hicks prefers to abandon the term "ratio of utilities" for his own phrase, the marginal rate of substitution:

> ...We may define the marginal rate of substitution of X for Y as the quantity of Y which would just compensate the consumer for the loss of a marginal unit of X.[10]

Through this device we eliminate the problem of cardinal measurement. The concept is particularly significant when we wish to describe an equilibrium with respect to a system

---

[10]Hicks, Value, p. 20.

of market prices. At this point a consumer's marginal rate of substitution between two goods will equal the ratio of prices of these goods. Were this not so, a consumer would find it advantageous to substitute more units of one good for a marginal unit of another, therefore achieving a new equilibrium. In terms of the indifference map equilibrium is shown by the tangency of a single curve and the so-called "price-line."

Hicks goes on to say that if we are to attack the marginal utility device, we must also destroy the principle upon which it is founded, diminishing marginal utility. But what can we substitute for it?

> ...By the rule that the indifference curves must be convex to the axes. This may be called...the principle of <u>diminishing marginal rate of substitution</u>.[11]

Indifference curve convexity will dictate the following:

> ...Suppose we start with a given quantity of goods, and then go on increasing the amount of X and diminishing that of Y in such a way that the consumer is left neither better off nor worse off on balance; then the amount of Y which has to be subtracted in order to set off a second unit of X will be less than that which has to be subtracted in order to set off the first unit. In other words, the more X is substituted for Y, the less will be the marginal rate of substitution of X for Y.[12]

This is not the same as diminishing marginal utility,

---

[11] <u>Ibid</u>.

[12] <u>Ibid</u>., pp. 20-21.

although both concepts are employed for similar reasons: that is, to define an equilibrium condition.

> ...Unless, at the point of equilibrium, the marginal rate of substitution is diminishing, equilibrium will not be stable.[13]

Hence, we can have a situation develop in which the marginal rate of substitution will be equal to the price ratio, and still not have an equilibrium. The reason is quite obvious: this identity can hold under conditions of both increasing and diminishing marginal rate of substitution. If the latter case is in effect, and the identity holds, the equilibrium must necessarily be a stable one. But if the marginal rate of substitution is increasing, and (again) the identity holds, consumers will find it advantageous to acquire a larger quantity of commodity; therefore, they would move to a new equilibrium position:

> ...It is clear, therefore, that for any point to be a possible rate of equilibrium at appropriate prices the marginal rate of substitution at that point must be diminishing.[14]

These concepts can best be clarified by a diagram, in which AB represents a consumer's indifference curve and ML, his price line:

---

[13] Ibid., p. 21.

[14] Ibid., p. 22.

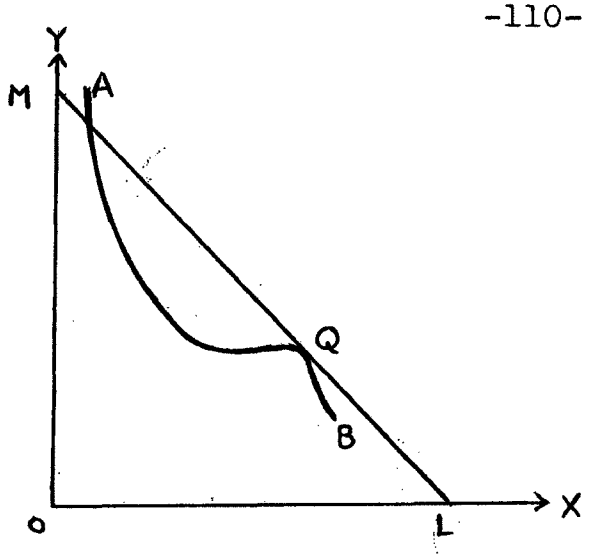



FIGURE IV

The reader will observe that at point Q there is an equation of the marginal rate of substitution and the price-ratio, because of the tangency of the indifference curve and the price-line. However, this cannot be a stable equilibrium, since at this point the marginal rate of substitution is increasing; that is, the indifference curve is concave to the axis. This implies that by moving to the right or left of Q the consumer is able to reach a higher indifference curve, thereby disturbing the original equilibrium. He would proceed until he reached another point at which a diminishing marginal rate of substitution was equal to the price ratio, and would remain there until some exogenous change brought about an increase in real income, allowing him to move to a higher indifference curve.

While the writings of economists of the fourth stage represent a definite advance in the literature, they have come under close scrutiny in the past few years. We are now in the process of approaching a fifth stage in the

evolution of demand theory. The outlines of this new approach are sketched in a number of journal articles[15] and a very few books published during the last two decades. Included in the latter category is Hicks' Revision of Demand Theory, which is his attempt at revising certain parts of Value and Capital.

The essential distinction between Hicks' new approach and that of earlier economists is that it takes advantage of econometric method and the so-called "preference hypothesis."

> ...Faced with factual data about quantities of commodities purchased and with the task of

---

[15]For further information see: F. Machlup, "Professor Hicks' Revision of Demand Theory," American Economic Review, XLVII, March, 1957.

F. Machlup, "The Problem of Verification in Economics," Southern Economic Journal, XXII, July, 1955.

W. E. Armstrong, "Theory of Consumers' Behavior," Oxford Economic Papers, 1950.

P. K. Newman, "The Foundations of Revealed Preference Theory," Oxford Economic Papers, June, 1955.

H. S. Houthakker, "Revealed Preference and the Utility Function," Economica, XVII, 1950.

O. Lange, "Complementarity and Inter-relations of Shifts in Demand," Review of Economic Studies, VIII, October, 1940.

S. Ichimura, "A Critical Note on the Definition of Related Goods," Review of Economic Studies, XVIII, 1950-51.

P. A. Samuelson, "Consumption Theorems in Terms of Over-compensation rather than Indifference Comparisons," Economica, XX, 1953.

> explaining changes in these quantities, the economist has at least three possibilities: explanations in terms of nonprice data, explanations in terms of effects of current price changes, and explanations in terms of lagged effects of price changes.[16]

What is needed is some method of isolating current-price effects from the other effects:

> ...The econometric purpose of the theory of demand is to give assistance in making this separation....The kind of theory which is needed... is one which will tell us something about the ways in which consumers would be likely to react if variations in current prices and incomes were the only causes of changes in consumption.[17]

This theory necessitates postulating an "ideal consumer," who, by definition, is unaffected by anything other than current market conditions. Using this postulate we are able to formulate certain principles of consumer behavior:

> ...The assumption of behavior according to a scale of preferences comes in here as the simplest hypothesis....What (is meant) by action according to a scale of preferences is the following. The ideal consumer...chooses that alternative, out of the various alternatives open to him, which he most prefers, or ranks most highly. In one set of market conditions he makes one choice, in others other choices; but the choices he makes always express the same ordering, and must therefore be consistent with one another.[18]

---

[16] Machlup, "Professor Hicks' Revision of Demand Theory," p. 120.

[17] J. R. Hicks, A Revision of Demand Theory, Oxford, Clarendon Press, 1956, p. 17.

[18] Ibid., pp. 17-18. Hicks admits that empirical testing of the preference hypothesis is not possible: "I feel obliged to conclude...that there is in practice no direct test of the preference hypothesis" (ibid., p. 58).

Economists must take care to distinguish between the two general types of logical ordering: "strong ordering," in which each commodity has a special, unique place in the order; and "weak ordering," in which certain commodities cluster in groups, and it becomes impossible to place one ahead of another. (An example of the latter is the indifference curve, since any point along a single curve is equally desirable as all other points.) When commodities are available only in discrete units, we must acknowledge the superiority of strong ordering. But "strong ordering has to be given up" whenever commodities are imperfectly divisible and the money used to purchase these goods is "finely divisible." Hicks prefers the latter approach, but admits that acceptance of it requires us to adopt two basic assumptions:

> ...that the consumer will always prefer a larger amount of money to a smaller amount and that his preference order is transitive.[19]

The latter assumption of transitivity implies that all items in a set are capable of being ordered in a "straightforward unidirectional manner."[20]

Once we accept the logic of weak ordering and the two additional assumptions, we are able to deduce all the major principles of the theory of consumer's demand. The

---

[19]Machlup, "Professor Hicks'," p. 121.

[20]Hicks, Revision, p. 27.

simplest case is, of course, that of the demand for a single commodity. Under this condition, a consumer is "...confronted with a market in which the price of no more than one good is liable to change."[21] His actions will depend on the principle of downward-sloping demand. We can prove this by isolating the substitution and income effects. The former will depend upon the so-called "consistency condition" (which is Hicks' term for the first assumption above). He demonstrates that this effect acts to increase the quantity demanded of a good when the good sells at a reduced price. Conversely, the income effect will depend on what Hicks' calls "observation" (Machlup prefers to define this effect in terms of "normal" and "inferior" goods, which amounts to the same thing).[22] When we deal with normal goods, the income and substitution effects of a change in price will be similar. But in the case of inferior goods--the so-called "Giffen Case"[23]--we have an exception to the law of downward-sloping demand. In this instance the substitution effect is much smaller than the income effect, and "the proportion of income spent upon the inferior good must be large."[24]

---

[21] Ibid., p. 47.

[22] See Machlup, "Professor Hicks," pp. 121-122.

[23] See Hicks, Revision, p. 67.

[24] Ibid., p. 66.

But economists are incorrect in assuming that the income and substitution effects can be nicely isolated as we have described. In actuality there is not one, but several pairs of effects, each referring to a different set of price variations. Delineation of these effects forms the largest part of Hicks' Revision of Demand Theory and cannot be summarized here. For further information the reader is referred to this work and others which have already been listed in the author's footnotes.

APPENDIX B

## Jevons' Theory of Distributive Shares: Selected References

The reader who wishes to investigate Jevons' theory of distribution further may find the following reference list of some value:

### BOOKS


Bell, J. F., A History of Economic Thought. New York: Ronald, 1953, pp. 414-416, 507, 575.

Davenport, H. J., Value and Distribution. Chicago: University of Chicago Press, 1908, pp. 209, 290-95.

Eckard, E. W., The Economics of W. S. Jevons. Washington, D. C.: American Council on Public Affairs, 1940, pp. 25-51.

Ferguson, J. M., Landmarks of Economic Thought. London: Longmans, 1938, pp. 161-162, 165, 182.

Howey, R. S., The Rise of the Marginal Utility School, 1870-1889. Lawrence, Kansas: University of Kansas Press, 1960, pp. 58-59.

Hutchison, T. W., A Review of Economic Doctrines, 1870-1929. Oxford: Clarendon Press, 1953, 44-46.

Jevons, W. S., The Principle of Economics. London: MacMillan, 1905, pp. 67-85, 98-120.

_____, The Theory of Political Economy. New York: Kelley & Millman, 1957, pp. 167-273.

Kinloch, T. F., Six English Economists. London: Gee, 1933, pp. 64-65.

Mitchell, W. C., Lecture Notes on Types of Economic Theory. New York: Kelley, 1949, pp. 41-46.

Roll, E., A History of Economic Thought. Englewood Cliffs, N. J.: Prentice-Hall, 1956, pp. 382-384.

Schumpeter, J. A., History of Economic Analysis. New York: Oxford University Press, 1954, pp. 637, 893, 902-903, 905, 913, 927-929, 940-942.

Stigler, G. J., Production and Distribution Theories. New York: MacMillan, 1941, pp. 13-37, 98.

## ARTICLES AND ESSAYS

Black, R. D., "Jevons and Cairnes," Economica, August, 1960, pp. 225-226.

Jevons, H. S., "William Stanley Jevons: His Scientific Contributions," Econometrica, July, 1934, pp. 234-236.

Jevons, W. S., "A Brief Account of a General Mathematical Theory of Political Economy," pp. 284-287.

Robertson, R. M., "Jevons and His Precursors," Econometrica, July, 1951, pp. 248-249.

## APPENDIX C

### Correspondence

In Appendix V (p. 333) to the Theory of Political Economy Jevons alludes to the fact that he employed intersecting supply and demand curves in his lectures at Owens College (1863). If this is correct, it would shed new light upon the question of priority in the graphic approach. Since none of Jevons' published works contain such supply-demand diagrams, the author has attempted to ascertain whether or not Jevons included these graphs in manuscripts which have not been reproduced. Letters of correspondence in this regard are duplicated below:

October 19, 1960

Dear Sirs:

As a student currently engaged in research into the writings of William Stanley Jevons, I would inquire whether any of his unpublished manuscripts are available through _____.

Mr. Augustus Kelley of Kelley & Millman, Inc., suggests that Jevons' lectures might have been privately reproduced and proposes that I contact you in this regard.

Yours truly,

D. P. Cole

********

The Library
University of Liverpool
Liverpool 3
November 11, 1960

Dear Mr. Cole,

I have been unable to trace a private reproduction of unpublished lectures of W. S. Jevons. This Library does not possess one, nor does our Dept. of Economics know of one. I have also enquired of the Bodleian Library, which keeps an index of photocopies held by British libraries, but they tell me that it records no manuscripts of any sort by Jevons.

I would suggest writing to the Department of Manuscripts, British Museum, London, W.C.I. They may be able to tell you where Jevons lectures are to be found.

Yours sincerely,

D. T. Cook
Curator of Special Collections

********

Department of Manuscripts
British Museum
London, W.C.I.
23rd November, 1960

Dear Sir,

The Keeper of Manuscripts has asked me to reply to your letter of 17th November.

No lecture notes of W. S. Jevons are listed in the published Catalogue of Additions to the Manuscripts in the British Museum. There is as yet no published index to MSS. acquired by this Department since 1930, but so far as I can see, they include no lecture notes of Jevons.

Yours faithfully,

M.S.F. Borrie
Assistant Keeper

# BIBLIOGRAPHY

BOOKS

Auspitz, R. and Lieben, R., Untersuchungen Über die Theorie des Preises. Leipzig: Duncker & Humblot, 1889.

Bell, J. F., A History of Economic Thought. New York: Ronald, 1953.

Bentham, J., A Fragment on Government and an Introduction to the Principles of Morals and Legislation. Oxford: Basil Blackwell, 1948.

Boehmert, W., W. Stanley Jevons und seine Bedeutung für die theoretische Nationalökonomie in England. Berlin: Privately printed, 1891.

Böhm-Bawerk, E. V., The Positive Theory of Capital. London: MacMillan, 1891.

Cornélissen, C., Théorie de la Valeur. Avec une Réfutation des Théories de Rodbertus, Karl Marx, Stanley Jevons & Boehm-Bawerk. Paris: Giard & Briere, 1913.

Davenport, H. J., Value and Distribution. Chicago: University of Chicago Press, 1908.

Eckard, E. W., The Economics of W. S. Jevons. Washington, D. C.: American Council on Public Affairs, 1940.

Edgeworth, F. Y., Mathematical Psychics. London: C. Kegan Paul, 1881.

Ferguson, J. M., Landmarks of Economic Thought. London: Longmans, 1938.

Gide, C. and Rist, C., History of Economic Doctrines. Boston: Heath, 1948.

Gossen, H., Entwickelung der Gesetze des menschlichen Verkehrs. Berlin: Prager, 1854.

Gray, A., The Development of Economic Doctrine. London: Longmans, 1931.

Haney, L. H., History of Economic Thought. New York: MacMillan, 1922.

Haney, L. H., Value and Distribution. New York: Appleton-Century, 1939.

Heilbroner, R. L., The Worldly Philosophers. New York: Simon and Schuster, 1953.

Heimann, E., History of Economic Doctrines. London: Oxford, 1945.

Hicks, J. R., A Revision of Demand Theory. Oxford: Clarendon Press, 1956.

_____, Value and Capital. Oxford: Clarendon Press, 1946.

Homan, P. T., Contemporary Economic Thought. New York: Harper, 1928.

Howey, R. S., The Rise of the Marginal Utility School, 1870-1889. Lawrence, Kansas: University of Kansas Press, 1960.

Hutchison, T. W., A Review of Economic Doctrines, 1870-1929. Oxford: Clarendon Press, 1953.

Jenkin, F., The Graphic Presentation of the Laws of Supply and Demand, and Other Essays on Political Economy. London: London School Reprints, 1931.

Jennings, R., Natural Elements of Political Economy. London: Longmans, 1855.

Jevons, H. A. (ed.) Letters and Journal of W. Stanley Jevons. London: MacMillan, 1886.

Jevons, W. S., The Coal Question. Cambridge: MacMillan, 1865.

_____, Investigations in Currency and Finance. London: MacMillan, 1884.

_____, Methods of Social Reform. London: MacMillan, 1883.

_____, Money and the Mechanism of Exchange. London: H. S. King, 1878.

_____, Primer of Political Economy. London: MacMillan, 1878.

_____, The Principles of Economics. London: MacMillan, 1905.

_____, The State in Relation to Labour. London: MacMillan, 1894.

_____, The Theory of Political Economy. New York: Kelley & Millman, 1957.

Keynes, J. M., Essays and Sketches in Biography. New York: Meridian, 1956.

Kinloch, T. F., Six English Economists. London: Gee, 1933.

La Nauze, J. A., Political Economy in Australia. Melbourne: Melbourne University Press, 1949.

Leftwich, R. H., The Price System and Resource Allocation. New York: Rhinehart, 1955.

Malthus, T., Principles of Political Economy. New York: Kelley, 1951.

Marshall, A., Elements of Economics of Industry. London: MacMillan, 1901.

_____, Industry and Trade. London: MacMillan, 1920.

_____, Money Credit and Commerce. London: MacMillan, 1924.

_____, Principles of Economics. London: MacMillan, 1898.

_____, The Pure Theory of Foreign Trade & The Pure Theory of Domestic Values. London: London School of Economics and Political Science, 1949.

Marx, K., Capital. New York: Modern Library, 1932.

Mayer, H., Die Wirtschaftstheorie der Gegenwart. Berlin: Prager, 1932.

McCulloch, J. R., Principles of Political Economy. Edinburgh: Black, 1849.

Mill, James, Elements of Political Economy. London: Longmans, 1821.

Mill, J. S., Principles of Political Economy. London: Longmans, 1936.

Mitchell, W. C., Lecture Notes on Types of Economic Theory. New York: Kelley, 1949.

Neff, F. A., Economic Doctrines. Wichita, Kansas: McGuin, 1946.

Newman, P. C., et al. (ed.) Source Readings in Economic Thought. New York: Norton, 1954.

Patterson, S. H., Readings in the History of Economic Thought. New York: McGraw-Hill, 1932.

Pigou, A. C., Alfred Marshall and Current Thought. London: MacMillan, 1953.

_____, (ed.) Memorials of Alfred Marshall. New York: Kelley & Millman, 1956.

Ricardo, D., Principles of Political Economy and Taxation. New York: MacMillan, 1921.

Robinson, J., The Economics of Imperfect Competition. London: MacMillan, 1942.

Roll, E., A History of Economic Thought. Englewood Cliffs, N. J.: Prentice-Hall, 1956.

Schumpeter, J. A., History of Economic Analysis. New York: Oxford University Press, 1954.

_____, Ten Great Economists. New York: Oxford University Press, 1951.

Scott, W. A., The Development of Economics. New York: Century, 1933.

Smith, A., An Inquiry into the Nature and Causes of the Wealth of Nations. New York: Modern Library, 1937.

Spann, O., The History of Economics. New York: Norton, 1930.

Spiegel, W., Development of Economic Thought. New York: Wiley, 1952.

Stigler, G. J., Production and Distribution Theories. New York: MacMillan, 1941.

Torrens, R., Essay on the Production of Wealth. London: Longmans, 1821.

Walras, L., Elements of Pure Economics. Homewood, Ill.: Irwin, 1954.

Wicksell, K., Über Wert, Kapital und Rente. London: London School Reprints, 1933.

## ARTICLES AND ESSAYS

Allen, R. G. D., Professor Slutsky's Theory of Consumer's Choice," Review of Economic Studies, IV, 1936.

Amoroso, L., "W. S. Jevons e la Economia Pura," Annali di Economia, II, November, 1925.

Armstrong, W. E., "Theory of Consumers' Behavior," Oxford Economic Papers, 1950.

Black, R. D., "Jevons and Cairnes," Economica, XXVII, August, 1960.

Hicks, J. R. and Allen, R. G. D., "A Reconsideration of the Theory of Value," Economica, III, 1934.

Higgins, B. H., "W. S. Jevons: A Centenary Estimate," Manchester School, VI, June, 1935.

Houthtaker, H. S., "Revealed Preference and the Utility Function," Economica, XVII, 1950.

Ichimura, S., "A Critical Note on the Definition of Related Goods," Review of Economic Studies, XVIII, 1950-51.

Jevons, H. W., "William Stanley Jevons: His Life," Econometrica, II, July, 1934.

Jevons, H. S., "William Stanley Jevons: His Scientific Contributions," Econometrica, II, July, 1934.

Jevons, W. S., "A Brief Account of a General Mathematical Theory of Political Economy," Journal of the Statistical Society of London, XXIX, 1866.

_____, "The Future of Political Economy," Fortnightly Review, XX, December, 1876.

_____, "The Progress of the Mathematical Theory of Political Economy," Journal of the Statistical Society of London, XXXVII, 1874.

_____, "Richard Cantillon and the Nationality of Political Economy," Contemporary Review, XXXIX, 1881.

Johnson, W. S., "The Pure Theory of Utility Curves," Economic Journal, 1913.

Knight, F. H., "Commonsense of Political Economy," Journal of Political Economy, XLII, 1934.

La Nauze, J. S., "Jevons in Sydney," Economic Record, XVII, 1941.

Lange, O., "Complementarity and Inter-relations of Shifts in Demand," Review of Economic Studies, VIII, October, 1940.

Machlup, F., "The Problem of Verification in Economics," Southern Economic Journal, XXII, July, 1955.

_____, "Professor Hicks' Revision of Demand Theory," American Economic Review, XLVII, March, 1957.

Newman, P. K., "The Foundations of Revealed Preference Theory," Oxford Economic Papers, June, 1955.

Robbins, L., "The Place of Jevons in the History of Economic Thought," Manchester School, VII, 1935.

Robertson, R. M., "Jevons and His Precursors," Econometrica, XIX, July, 1951.

Samuelson, P. A., "Consumption Theorems in Terms of Overcompensation rather than Indifference Comparisons," Economica, XX, 1953.

Slutsky, E., "Sulla Theoria del Bilancio del Consummatore," Giornale degli Economisti, LI, 1915.

Stigler, G. J., "The Development of Utility Theory," Journal of Political Economy, LVIII, 1950.

_____, "Ricardo and the 93% Labor Theory of Value," American Economic Review, XLVIII, June, 1958.

Wicksteed, P. H., "On Certain Passages in Jevons' Theory of Political Economy," Quarterly Journal of Economics, III, 1889.

_____, "Stanley Jevons," Commonsense of Political Economy, II, 1933.

Young, A. A., "Jevons' Theory of Political Economy," American Economic Review, II, 1912.